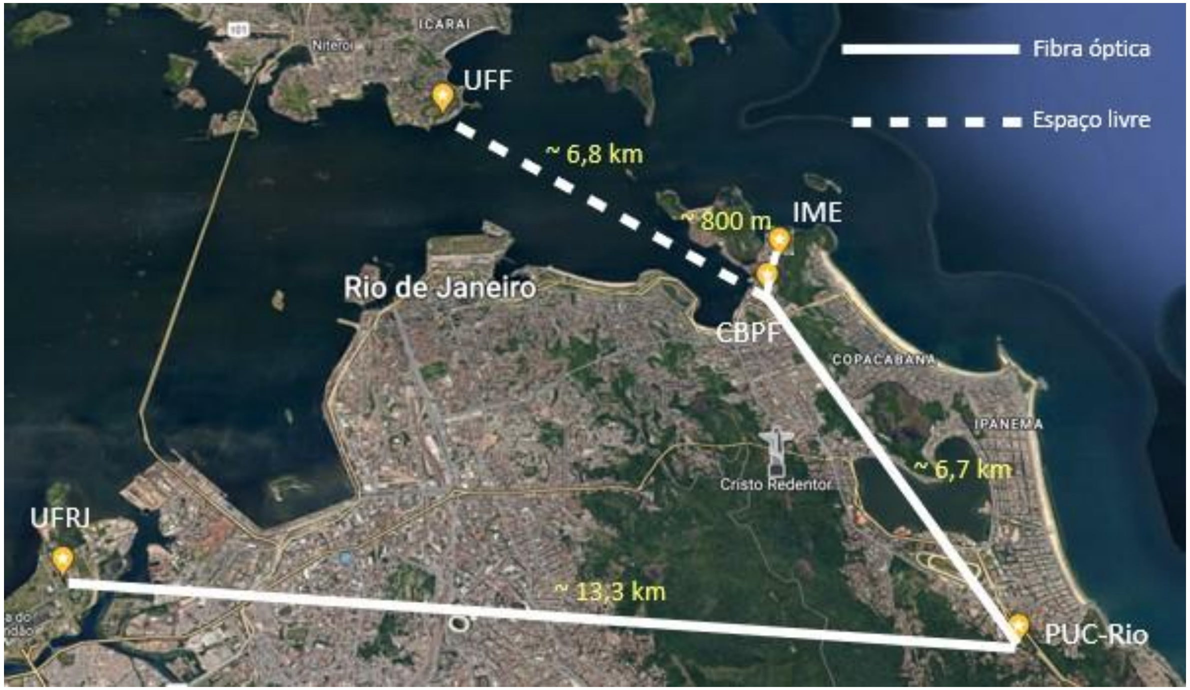
Recognition of light orbital angular momentum superpositions for quantum communication

INSTITUTO DE FÍSICA
Universidade Federal Fluminense

Antonio Zelaquett Khoury

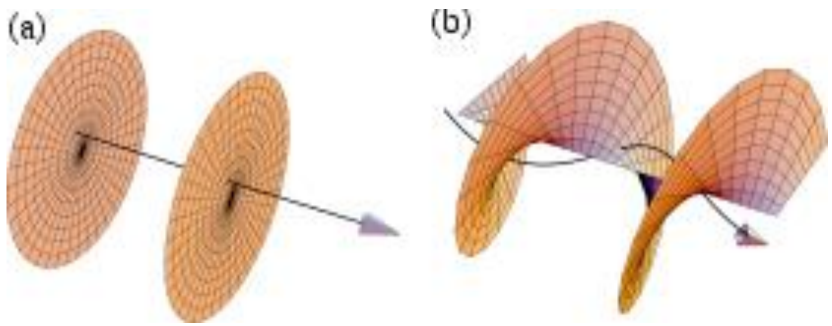


QOART – January 21st, 2025

Structured light and applications

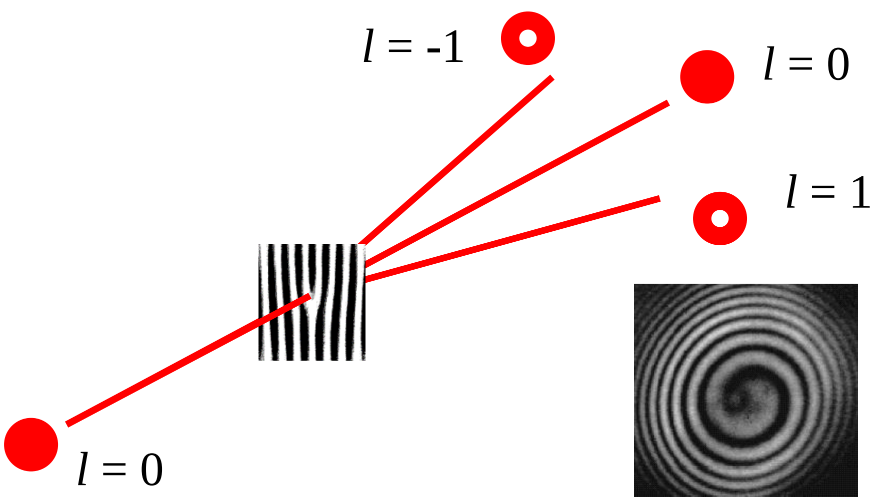
Light Orbital Angular Momentum

Twisted wavefront



kz=0

kz+lϕ=0



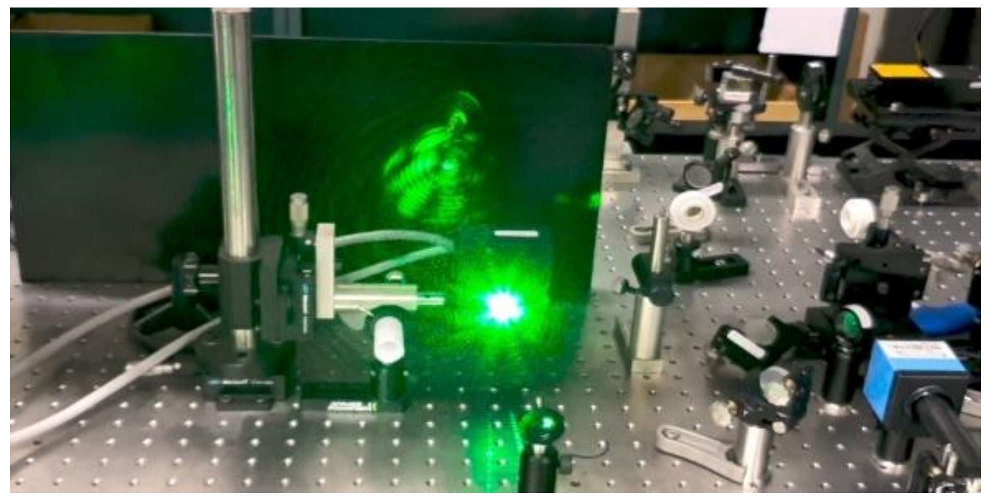
Angular momentum

$$\mathbf{J} = \mathbf{J}_S + \mathbf{J}_O$$

SPIN + ORBITAL

pol

wavefront



Spatial Light Modulator

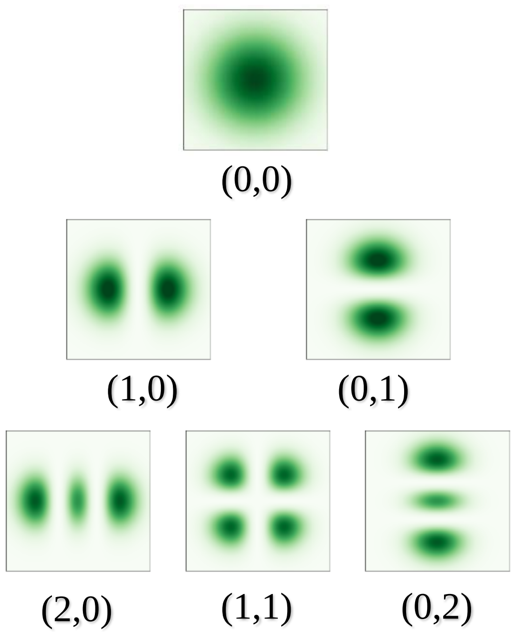
General Paraxial Modes

Paraxial Equation

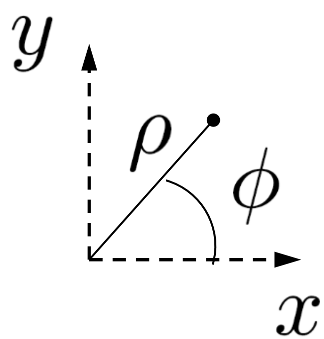
$$\nabla_{\perp}^{2}\psi + 2ik\frac{\partial \psi}{\partial z} = 0$$

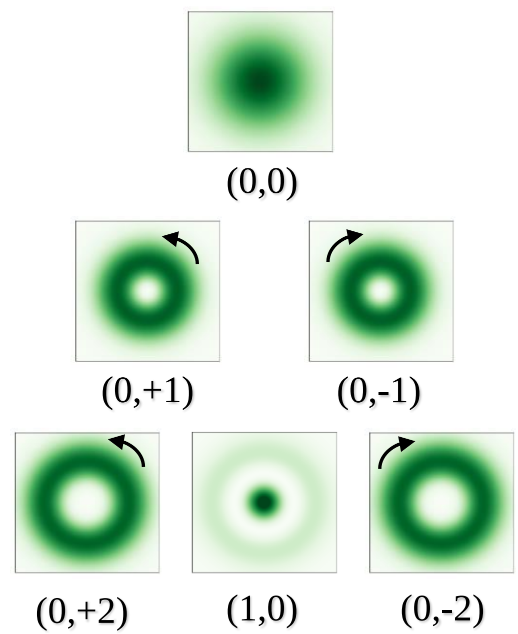
Hermite-Gauss (HG)

Laguerre-Gauss (LG)



Rectangular	**Cylindrical**
$HG_{nm}(x,y)$	$LG_{pl}(\rho,\phi)$





Hermite-Gaussian Modes

$$HG_{mn}(\mathbf{r}) = \frac{A_{mn}}{w(z)} H_m\left(\frac{\sqrt{2}x}{w(z)}\right) H_n\left(\frac{\sqrt{2}y}{w(z)}\right) \exp\left(-\frac{x^2+y^2}{w^2(z)}\right) \exp\left(ik\frac{x^2+y^2}{2R(z)}\right) e^{-i\varphi_N(z)}$$

Gouy phase

$$\varphi_N(z) = (N+1)\arctan\left(z/z_R\right)$$

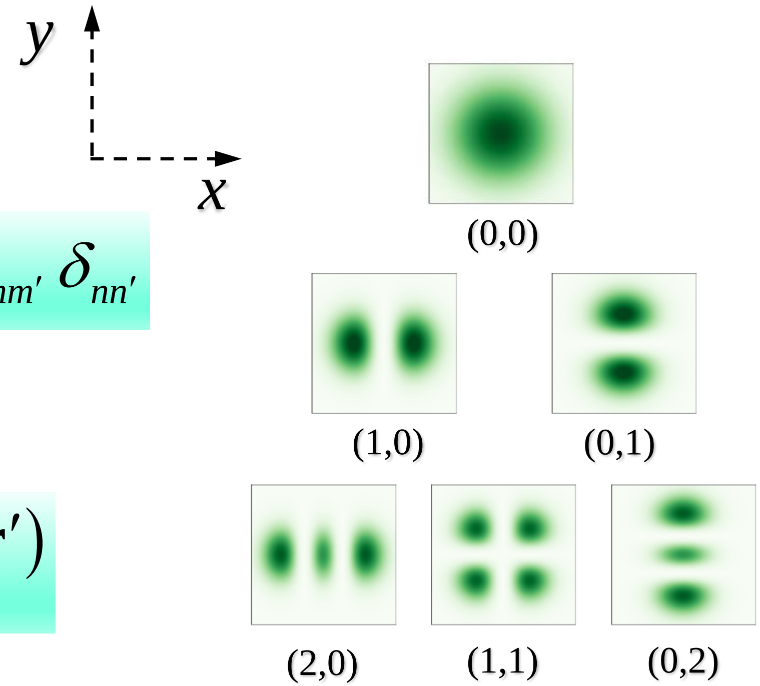
$N=m+n$

Orthonormal

$$\int HG^*_{mn}(\mathbf{r})\, HG_{m'n'}(\mathbf{r})\, d^2\mathbf{r} = \delta_{mm'}\,\delta_{nn'}$$

Complete

$$\sum_{m,n} HG^*_{mn}(\mathbf{r})\, HG_{mn}(\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}')$$



$N=0$

$N=1$

$N=2$

Hermite-Gaussian Modes

$$HG_{mn}(\mathbf{r}) = \frac{A_{mn}}{w(z)} H_m\left(\frac{\sqrt{2}x'}{w(z)}\right) H_n\left(\frac{\sqrt{2}y'}{w(z)}\right) \exp\left(-\frac{x'^2+y'^2}{w^2(z)}\right) \exp\left(ik\frac{x'^2+y'^2}{2R(z)}\right) e^{-i\varphi_N(z)}$$

Gouy phase

$$\varphi_N(z) = (N+1)\arctan\left(z/z_R\right)$$

N=m+n

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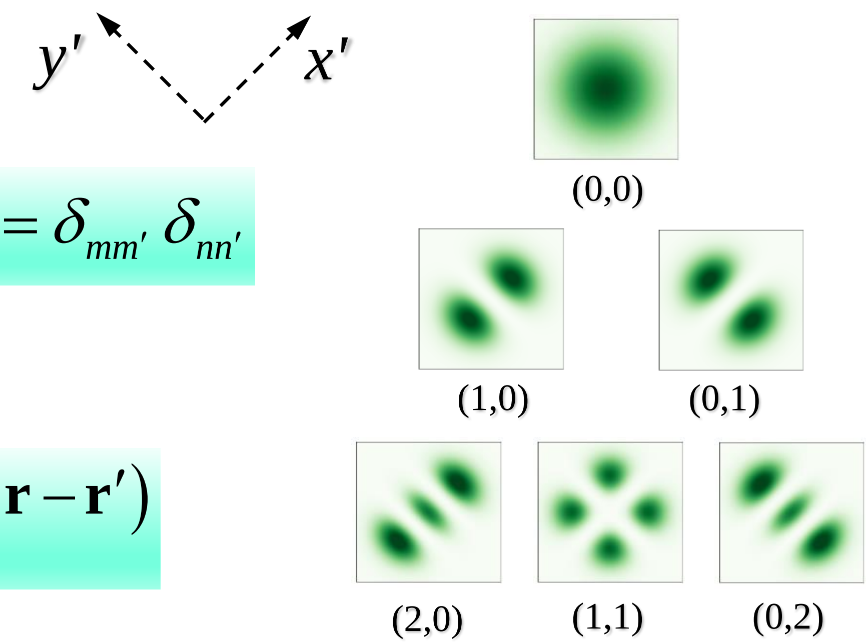
$$\int HG^*_{mn}(\mathbf{r})\, HG_{m'n'}(\mathbf{r})\, d^2\mathbf{r} = \delta_{mm'}\,\delta_{nn'}$$

Complete

$$\sum_{m,n} HG^*_{mn}(\mathbf{r})\, HG_{mn}(\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}')$$



N=0

N=1

N=2

Laguerre-Gaussian Modes

$$LG_{pl}(\mathbf{r}) = \frac{A_{pl}}{w(z)}\left(\frac{\sqrt{2}\rho}{w(z)}\right)^{|l|} L_p^{|l|}\left(\frac{2\rho^2}{w^2(z)}\right)\exp\left(-\frac{\rho^2}{w^2(z)}\right)\exp\left(ik\frac{\rho^2}{2R(z)}\right)e^{-i\varphi_N(z)}e^{il\phi}$$

Gouy phase

$$\varphi_N(z) = (N+1)\arctan\left(z/z_R\right)$$

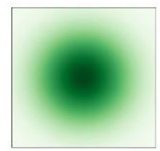
$N=2p+|l|$

Orthonormal

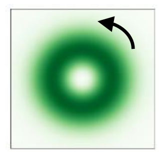
$$\int LG^*_{pl}(\mathbf{r})\,LG_{p'l'}(\mathbf{r})\,d^2\mathbf{r} = \delta_{pp'}\,\delta_{ll'}$$

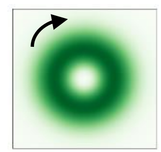
Complete

$$\sum_{p,l} LG^*_{pl}(\mathbf{r})\,LG_{pl}(\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}')$$

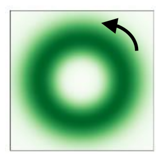

(0,0)

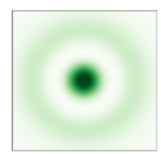
$N=0$

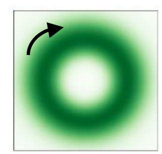

(0,+1)


(0,-1)

$N=1$


(0,+2)


(1,0)


(0,-2)

$N=2$

Algebraic structure of paraxial wave functions

$$LG_{pl}(\mathbf{r}) = \sum_{k=0}^{N} \alpha_k HG_{k,N-k}(\mathbf{r})$$

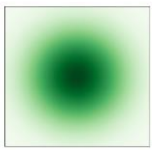
E. Abramochkin and V. Volostnikov,
Opt. Commun. 83, 123 (1991)

LG-HG Unitary transformation

$$U_{mn}^{pl} = \sum_{\oplus} U^N$$

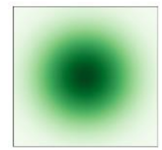
Hermite-Gauss (HG)

Laguerre-Gauss (LG)



(0,0)

$\leftarrow SU(1) \rightarrow$

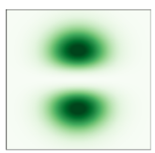


(0,0)

$\psi_H(\mathbf{r})$

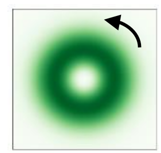
$\psi_V(\mathbf{r})$

(1,0)

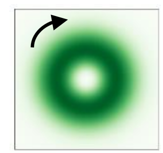


(0,1)

$\leftarrow SU(2) \rightarrow$



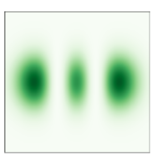
(0,+1)



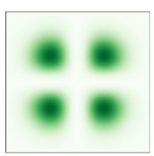
(0,-1)

$\psi_+(\mathbf{r})$

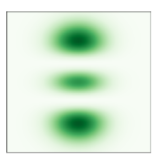
$\psi_-(\mathbf{r})$



(2,0)

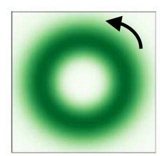


(1,1)

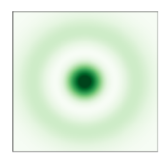


(0,2)

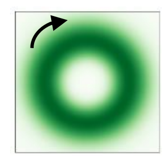
$\leftarrow SU(3) \rightarrow$



(0,+2)



(1,0)



(0,-2)

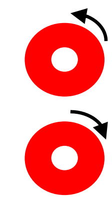
Astigmatic mode transformations

$$\psi_H(\mathbf{r}) \propto x\, e^{-(x^2+y^2)/w^2}$$

$$\psi_V(\mathbf{r}) \propto y\, e^{-(x^2+y^2)/w^2}$$

HG-LG

$$\psi_\pm = \frac{\psi_H \pm i\psi_V}{\sqrt{2}}$$

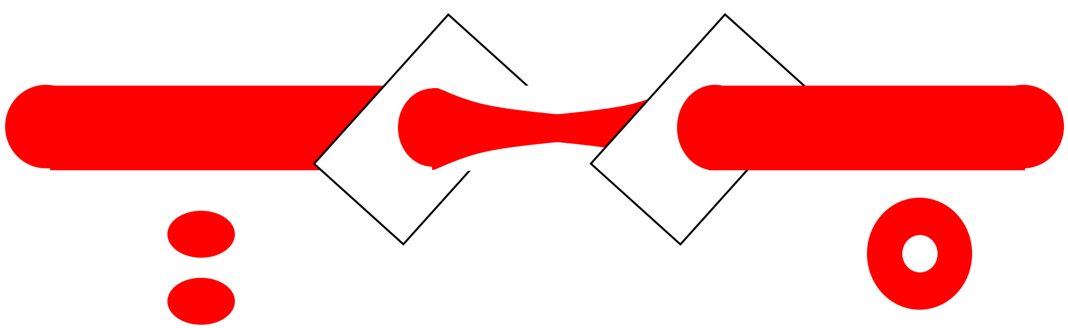


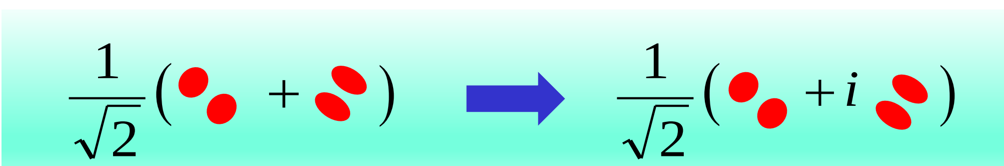
HG-HG

$$\psi_{\pm 45^\circ} = \frac{\psi_H \pm \psi_V}{\sqrt{2}}$$

Mode Converter

cylindrical lenses at 45º



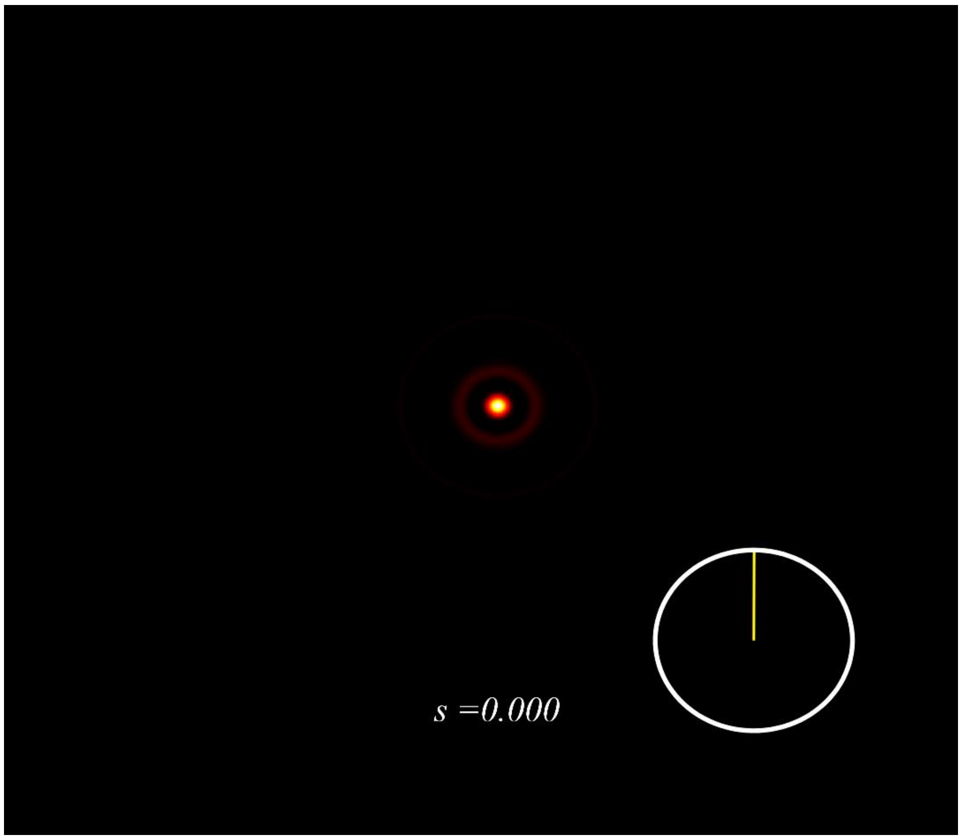


MC eingenvectors

$$\psi_{\pm 45^\circ} = \frac{\psi_H \pm \psi_V}{\sqrt{2}}$$

Self-Restoring Spot

$$\psi(z) = 0.3LG_{0,0}(\mathbf{r}) + LG_{6,0}(\mathbf{r}) + LG_{12,0}(\mathbf{r})$$



Gouy phase

$$\varphi_N(z) = (N+1)s(z)\frac{\pi}{2}$$

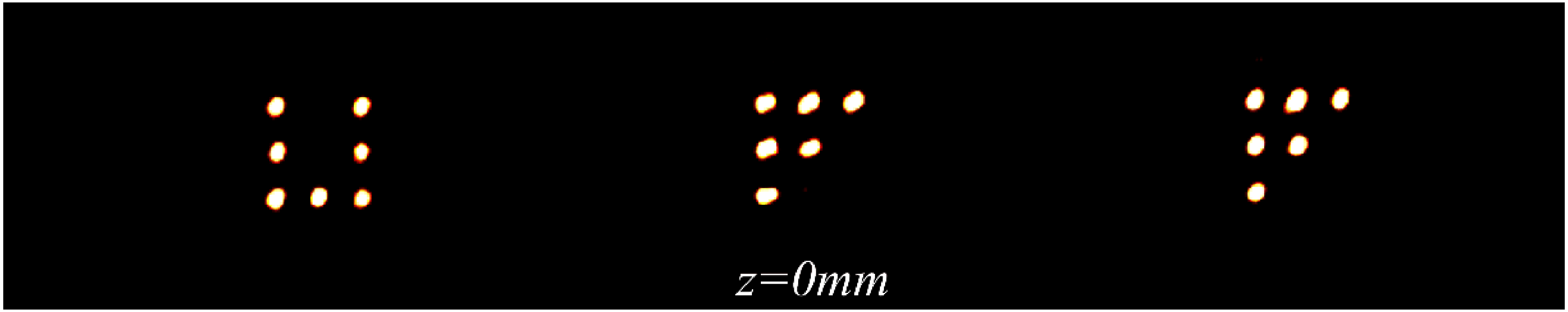
$$s(z) = \frac{2}{\pi}\arctan\left(\frac{z}{z_R}\right)$$

$$0 < z < \infty \Rightarrow 0 < s(z) < 1$$

Synchronisation

$$s(z) = \frac{4q}{N_2 - N_1} \qquad (q \in \mathbf{N})$$

Self-Restoring Spot Array

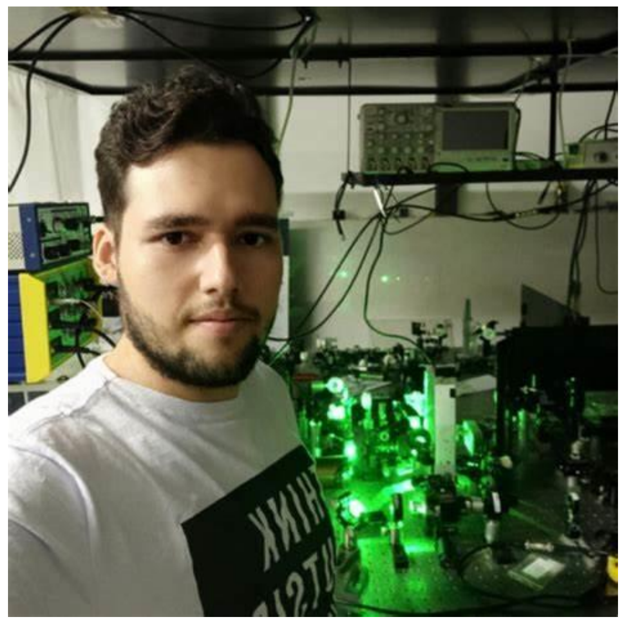


B. Pinheiro da Silva et al,
Phys. Rev. Lett. 124, 033902 (2020)

Application

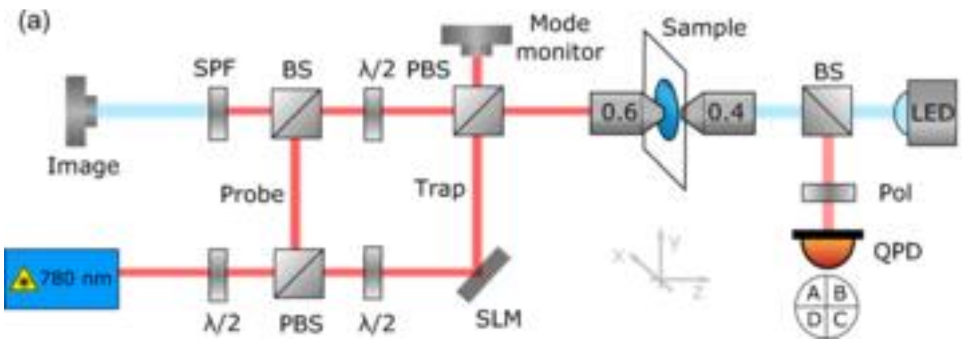
Dark Optical Tweezer (PUC-UFF)

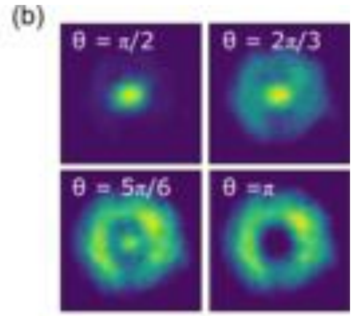
Phys. Rev. Lett. 131, 163601 (2023)

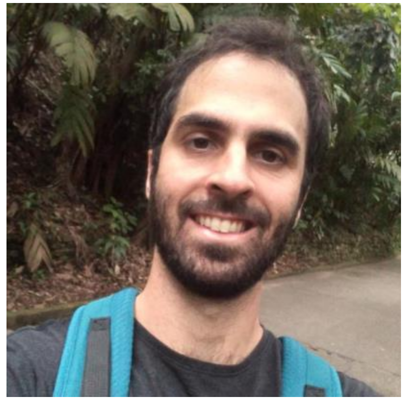


Trapping in a dark focus

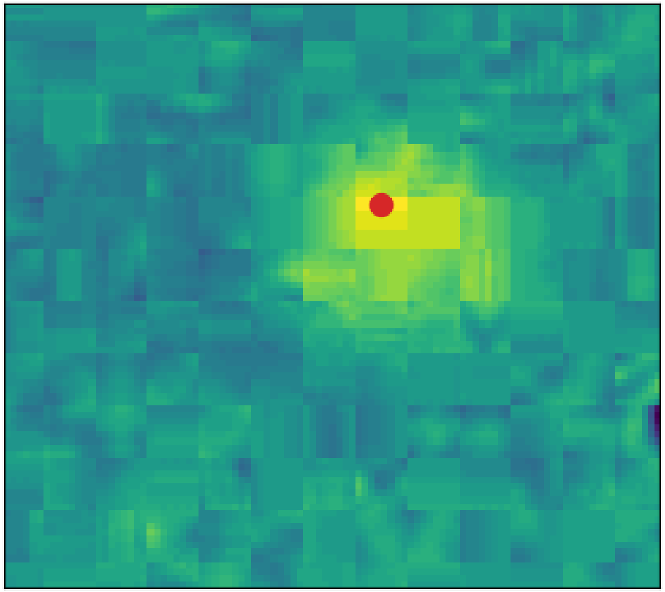
Phys. Rev. Lett. 131, 163601 (2023)

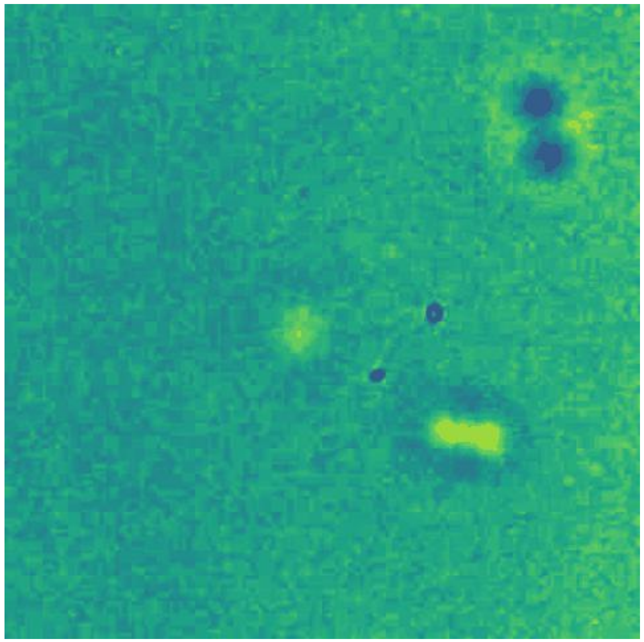






Thiago Guerreiro
PUC-Rio





Spin-Orbit non separability

Algebraic structure of paraxial wave functions

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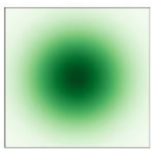
E. Abramochkin and V. Volostnikov,
Opt. Commun. 83, 123 (1991)

LG-HG Unitary transformation

$$U_{mn}^{pl} = \sum_{\oplus} U^{N}$$

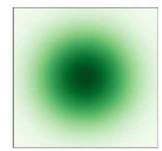
Hermite-Gauss (HG)

Laguerre-Gauss (LG)



(0,0)

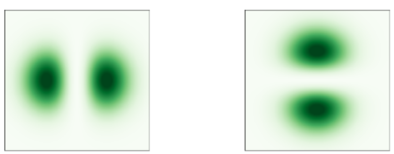
← SU(1) →



(0,0)

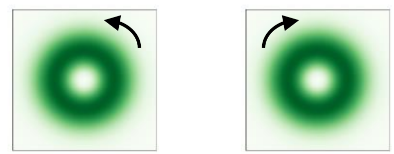
$\psi_H(\mathbf{r})$

$\psi_V(\mathbf{r})$



(1,0) (0,1)

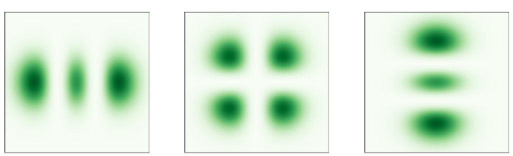
← SU(2) →



(0,+1) (0,-1)

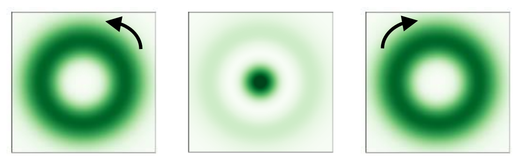
$\psi_+(\mathbf{r})$

$\psi_-(\mathbf{r})$



(2,0) (1,1) (0,2)

← SU(3) →

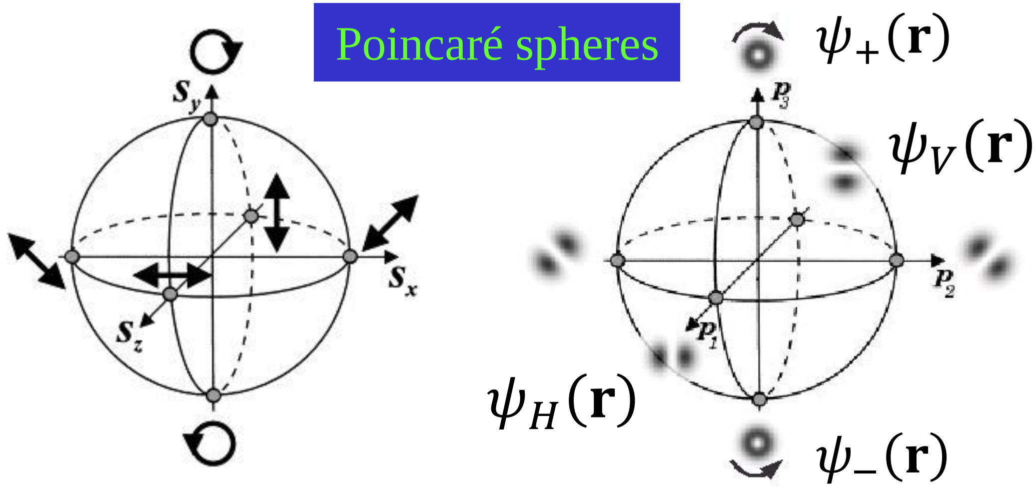


(0,+2) (1,0) (0,-2)

Spin-orbit nonseparable modes

Poincaré spheres

POLARIZATION



SPATIAL

Tensor product in CO

$$\Psi_{sep} = \phi(\mathbf{r}) \otimes \hat{\xi}$$

(spatial ⊗ polarization)

$$\Psi_{ent} \neq \phi(\mathbf{r}) \otimes \hat{\xi} \quad \Rightarrow$$

$$\Psi^{\pm} = \frac{1}{\sqrt{2}}\left[\psi_H(\mathbf{r})\hat{\mathbf{e}}_H \pm \psi_V(\mathbf{r})\hat{\mathbf{e}}_V\right]$$

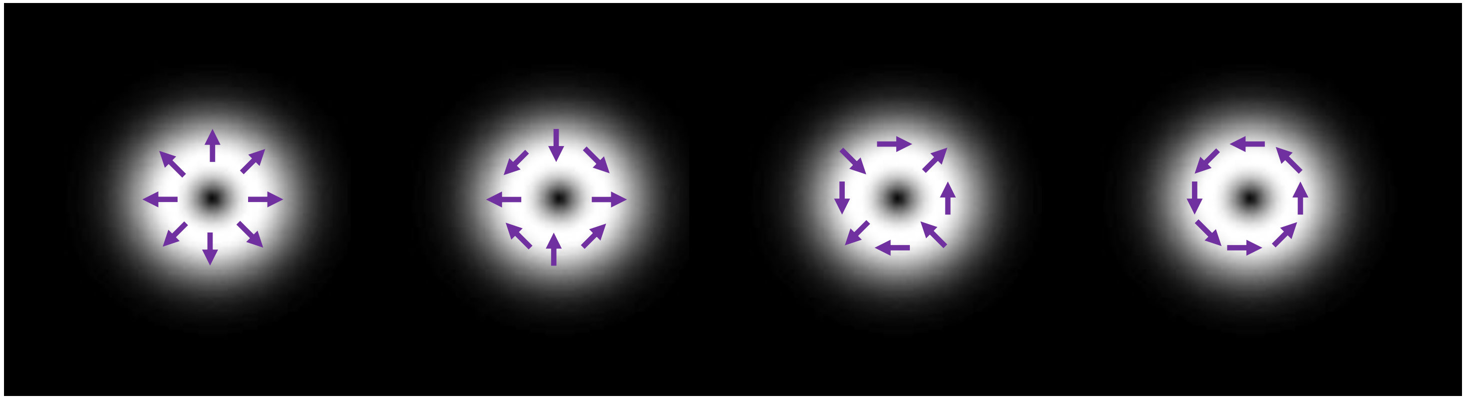
$$\Phi^{\pm} = \frac{1}{\sqrt{2}}\left[\psi_H(\mathbf{r})\hat{\mathbf{e}}_V \pm \psi_V(\mathbf{r})\hat{\mathbf{e}}_H\right]$$

Bell modes

$$\Psi = \alpha\,\psi_H(\mathbf{r})\hat{\mathbf{e}}_H + \beta\,\psi_H(\mathbf{r})\hat{\mathbf{e}}_V + \gamma\,\psi_V(\mathbf{r})\hat{\mathbf{e}}_H + \delta\,\psi_V(\mathbf{r})\hat{\mathbf{e}}_V$$

$$C = 2|\alpha\delta - \beta\gamma|$$

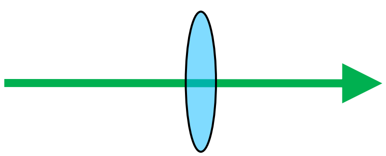
Polarization vortices



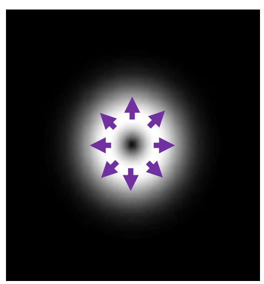
$\boldsymbol{\Psi}^{+}$ $\boldsymbol{\Psi}^{-}$ $\boldsymbol{\Phi}^{+}$ $\boldsymbol{\Phi}^{-}$

$$\boldsymbol{\Psi}^{\pm} = \frac{1}{\sqrt{2}}\left[\psi_H(\mathbf{r})\,\hat{\mathbf{e}}_H \pm \psi_V(\mathbf{r})\,\hat{\mathbf{e}}_V\right]$$

$$\boldsymbol{\Phi}^{\pm} = \frac{1}{\sqrt{2}}\left[\psi_H(\mathbf{r})\,\hat{\mathbf{e}}_V \pm \psi_V(\mathbf{r})\,\hat{\mathbf{e}}_H\right]$$



S-Plate



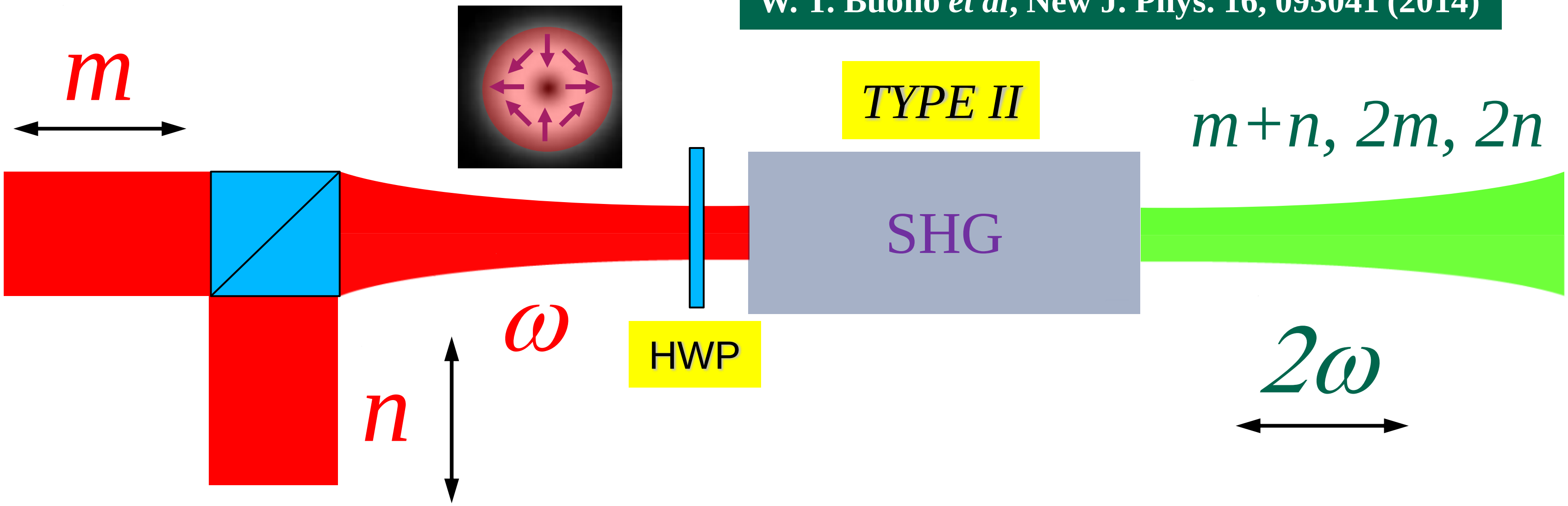
$\boldsymbol{\Psi}^{+}$

Spin-orbit coupling in liquid crystals

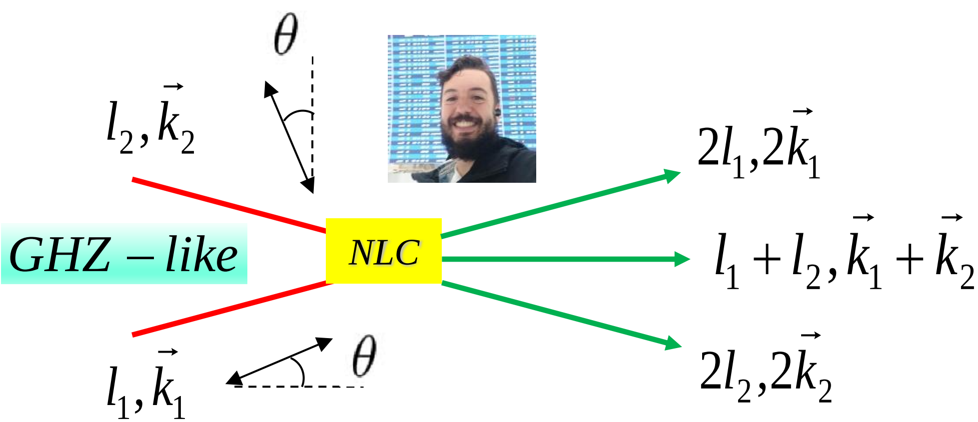
L. Marrucci, C. Manzo, and D. Paparo, Phys. Rev. Lett. 96, 163905 (2006)

Structured nonlinear optics

W. T. Buono *et al*, New J. Phys. 16, 093041 (2014)

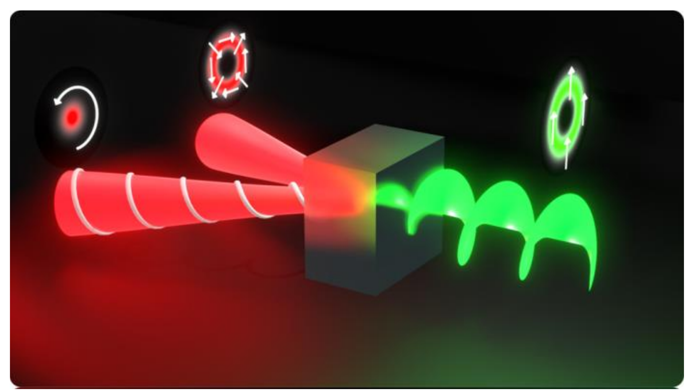


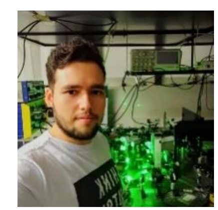
Polarization controlled OAM operations



W. T. Buono *et al*, Opt. Lett. 43, 1439 (2018)

Spin-to-OAM transfer





B. Pinheiro da Silva *et al*, Nanophot. 11, 771 (2022)

Structured Light Tomography

Statement of the problem

Given an arbitrary superposition of fixed order

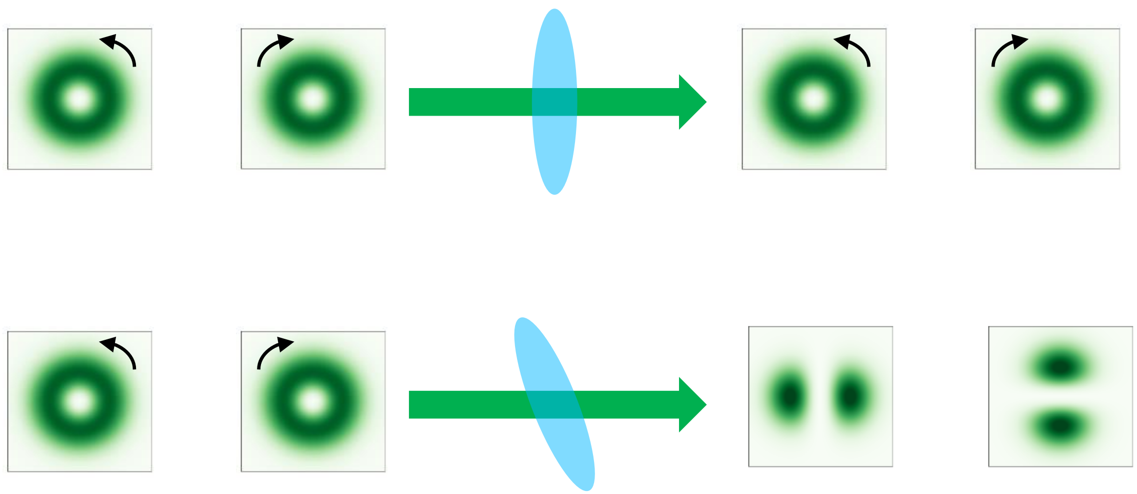
$$\psi(\mathbf{r}) = \sum_{p,l} \alpha_{pl}\, LG_{pl}(\mathbf{r}) \quad (2p + |l| = N \text{ and } \alpha \in \mathbf{C})$$

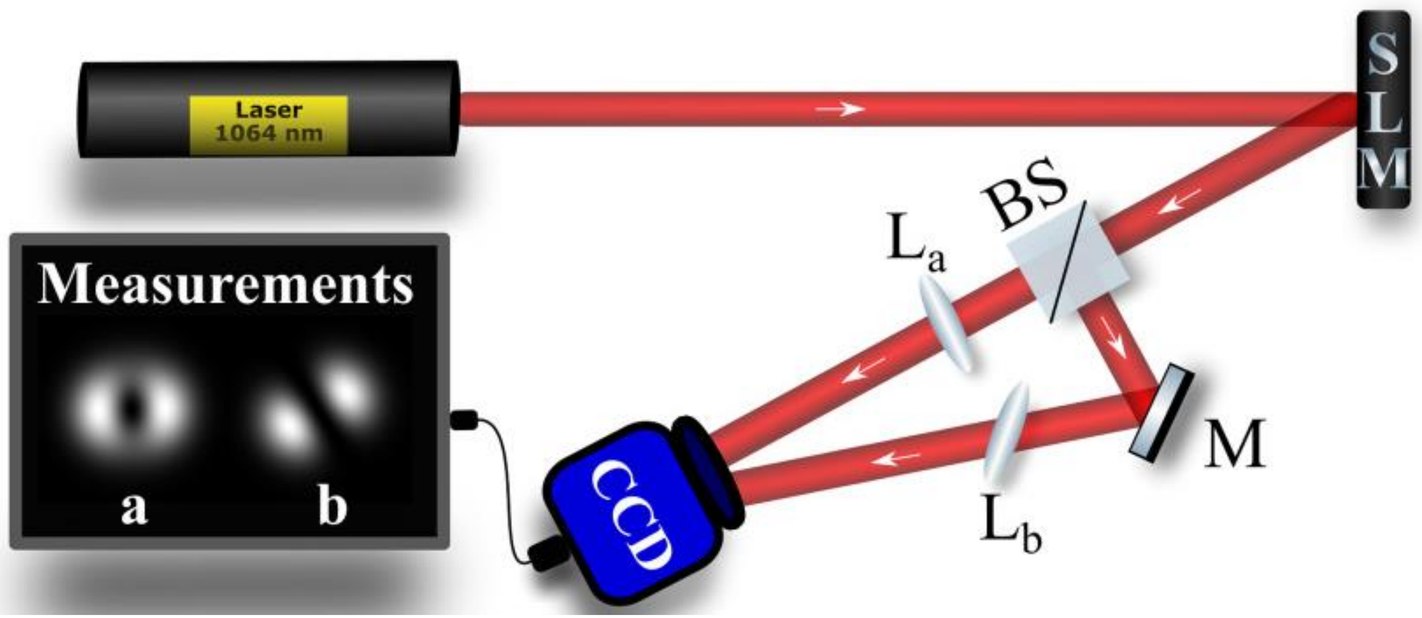
We wish to determine α_{pl} **from intensity distribution *only***

Problem:

$$\left| \sum_{p,l} \alpha_{pl}\, LG_{pl}(\mathbf{r}) \right|^2 = \left| \sum_{p,l} \alpha^*_{p,-l}\, LG_{pl}(\mathbf{r}) \right|^2$$

Symmetry lift by astigmatic transformation





Tomography of transverse mode superpositions

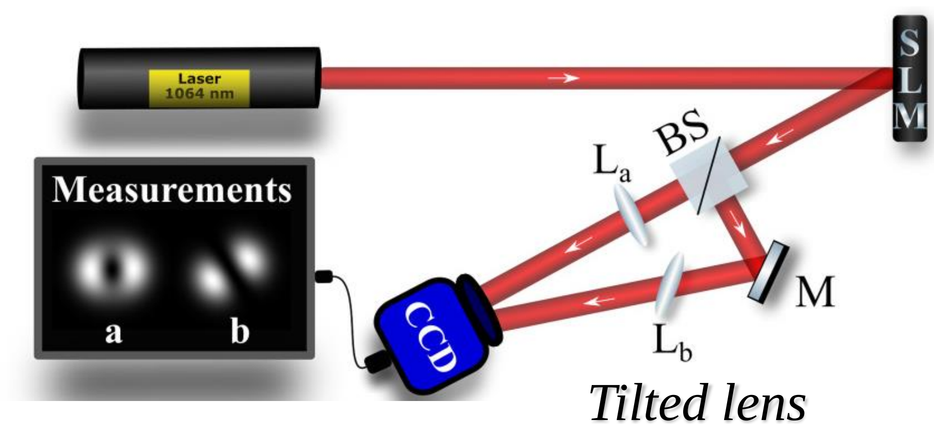
Let $\psi(\mathbf{r}) = \sum_{p,l} \alpha_{pl}\, LG_{pl}(\mathbf{r})$, we wish to determine $\{\alpha_{pl}\}$ from intensity measurements only.

Problem: $\left|\sum_{p,l} \alpha_{pl}\, LG_{pl}(\mathbf{r})\right|^2 = \left|\sum_{p,l} \alpha^*_{p,-l}\, LG_{pl}(\mathbf{r})\right|^2$

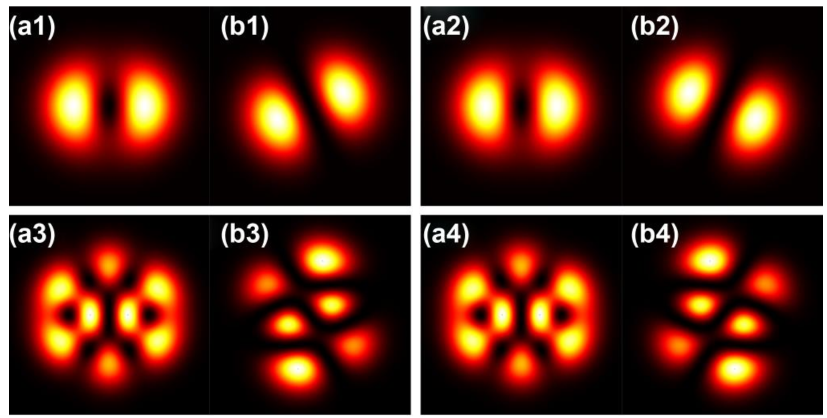
Solution: astigmatic image $\rightarrow$ tilted lens

$$\psi_1(\mathbf{r}) = 0.92\, LG_{0,+1}(\mathbf{r}) + 0.38\, LG_{0,-1}(\mathbf{r})$$
$$\psi_2(\mathbf{r}) = 0.38\, LG_{0,+1}(\mathbf{r}) + 0.92\, LG_{0,-1}(\mathbf{r})$$

$$\psi_3(\mathbf{r}) = 0.5\left[LG_{0,+3}(\mathbf{r}) - LG_{0,-3}(\mathbf{r}) + LG_{1,+1}(\mathbf{r}) + LG_{1,-1}(\mathbf{r})\right]$$
$$\psi_4(\mathbf{r}) = 0.5\left[-LG_{0,+3}(\mathbf{r}) + LG_{0,-3}(\mathbf{r}) + LG_{1,+1}(\mathbf{r}) + LG_{1,-1}(\mathbf{r})\right]$$



Tilted lens



B. Pinheiro da Silva *et* al, Phys. Rev. A 103, 063704 (2021)

Partially coherent beams

More generally, given a coherence function

$$\Psi(\mathbf{r},\mathbf{r}') = \sum_{i,j} \rho_{ij}\, u_i^*(\mathbf{r})\, u_j(\mathbf{r}')$$

We wish to determine ρ_{ij}

Gell-Mann matrices

$$X_{jk} = \frac{|u_j\rangle\langle u_k| + |u_k\rangle\langle u_j|}{\sqrt{2}}$$

$$Y_{jk} = \frac{i\left(|u_j\rangle\langle u_k| - |u_k\rangle\langle u_j|\right)}{\sqrt{2}}$$

$$Z_j = \frac{\sum_{r=1}^{j} |u_r\rangle\langle u_r| - j|u_{j+1}\rangle\langle u_{j+1}|}{\sqrt{j(j+1)}}$$

High dimensional tomography

$$\rho = \frac{\mathbf{1}}{d} + \sum_{jk} \alpha_{jk} X_{jk} + \beta_{jk} Y_{jk} + \sum_j \gamma_j Z_j$$

$$\alpha_{jk} \propto Tr\left(\rho X_{jk}\right),\ \beta_{jk} \propto Tr\left(\rho Y_{jk}\right),\ \gamma_j \propto Tr\left(\rho Z_j\right)$$

Far too difficult at high d

Intensity measurements

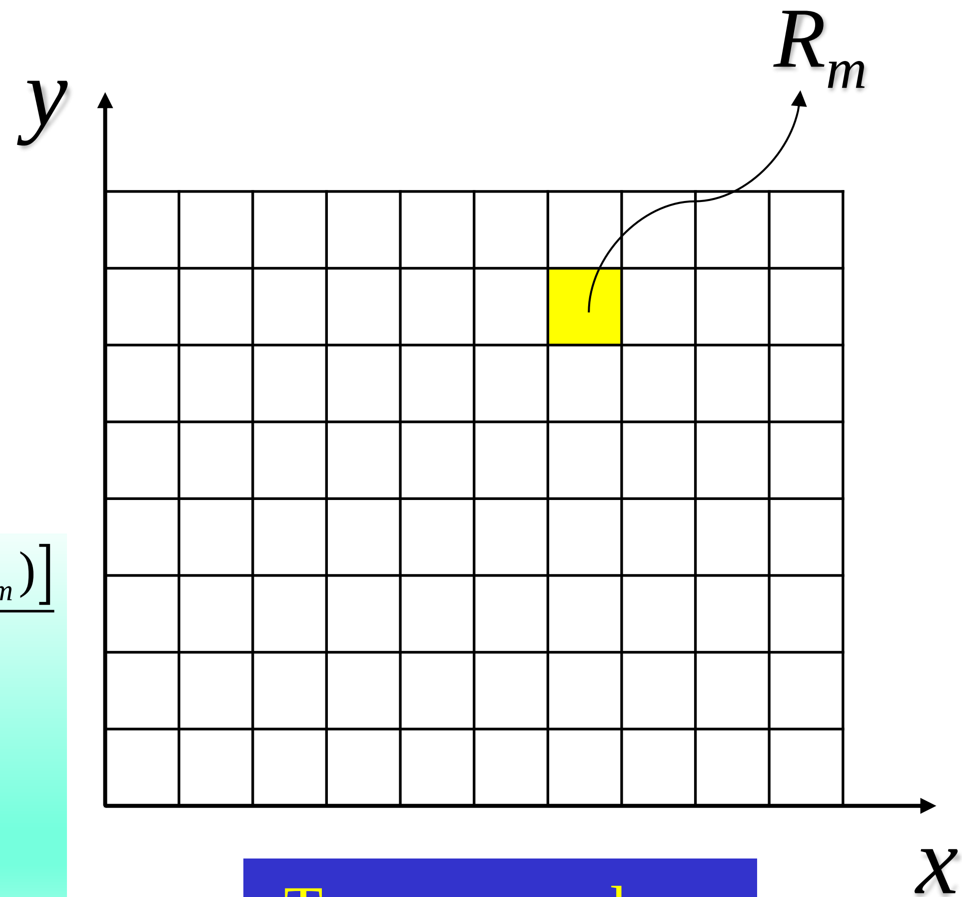
Position POVM

$$\left\{\Pi(R_1),\Pi(R_2)\cdots\Pi(R_m)\cdots\right\}$$

$$\Pi(R_m)=\int_{R_m}|\mathbf{r}\rangle\langle\mathbf{r}|d\mathbf{r} \quad\Rightarrow\quad \sum_{m=-\infty}^{+\infty}\Pi(R_m)=\mathbf{1}$$

$$p_m=Tr\left[\rho\Pi(R_m)\right] \quad\Rightarrow\quad q_m=p_m-\frac{Tr\left[\Pi(R_m)\right]}{d}$$

$$\mathbf{q}=\mathbf{T}\begin{bmatrix}\alpha_{jk}\\ \beta_{jk}\\ \gamma_j\end{bmatrix} \quad\Rightarrow\quad \begin{bmatrix}\alpha_{jk}\\ \beta_{jk}\\ \gamma_j\end{bmatrix}=\left(\mathbf{T}^\dagger\mathbf{T}\right)^{-1}\mathbf{T}^\dagger\mathbf{q}$$



Transverse plane

If T is injective, then the POVM is informationally complete

Tomography from intensity measurements

High dimensional tomography

$$\rho = \frac{\mathbf{1}}{d} + \sum_{jk} \alpha_{jk} X_{jk} + \beta_{jk} Y_{jk} + \sum_{j} \gamma_j Z_j$$

$$I(R_n) = Tr\left[\rho \Pi(R_n)\right]$$

$$I(R_n) = \frac{Tr\left[\Pi(R_n)\right]}{d} + \sum_{jk} Tr\left[\Pi(R_n) X_{jk}\right]\alpha_{jk} + Tr\left[\Pi(R_n) Y_{jk}\right]\beta_{jk} + \sum_{j} Tr\left[\Pi(R_n) Z_j\right]\gamma_j$$

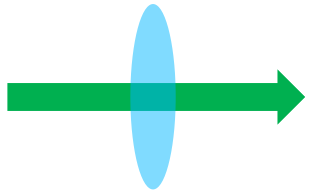
One equation for each pixel information! However…

$\left\{\Pi(R_j)\right\}$ is **not** informationally complete!

$$Tr\left[Y_{jk}\,\Pi(R)\right] = \sqrt{2}\,\mathrm{Im}\int_R u_j^*(\mathbf{r})\,u_k(\mathbf{r})\,d\mathbf{r} = 0 \quad \text{for } u_j, u_k \in \{HG\}$$

Informationally complete POVM

$$\{\Pi(R_j)\}$$



$$\{\Pi(R_j)\}$$

$$\{\Pi(R_j)\}$$

$$\{\Pi_\theta(R_j) = U_\theta \Pi(R_j) U_\theta^\dagger\}$$

$$U_\theta$$

Astigmatic transformation

$$U_\theta = e^{i(m-n)\theta} \sum_{m,n} |HG_{mn}\rangle\langle HG_{mn}|$$

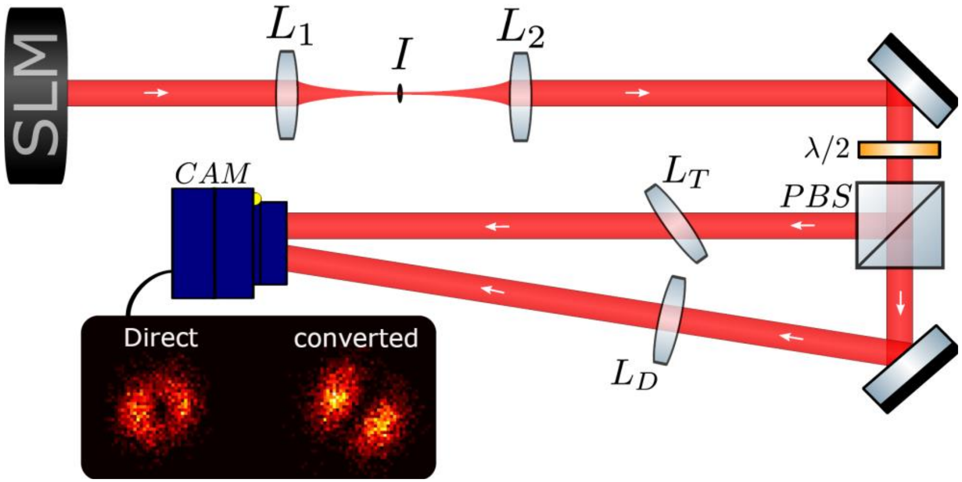
$$Tr\left[X_{jk}\Pi_\theta(R)\right] = \sqrt{2}\cos[(j-k)\theta]\,Tr\left[X_{jk}\Pi(R)\right]$$

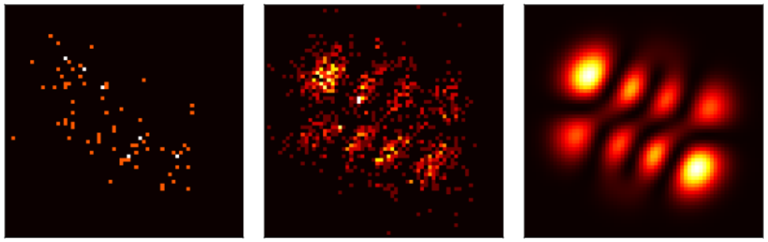
$$Tr\left[Y_{jk}\Pi_\theta(R)\right] = \sqrt{2}\sin[(j-k)\theta]\,Tr\left[X_{jk}\Pi(R)\right]$$

$$Tr\left[Z_j\Pi_\theta(R)\right] = Tr\left[Z_j\Pi(R)\right]$$

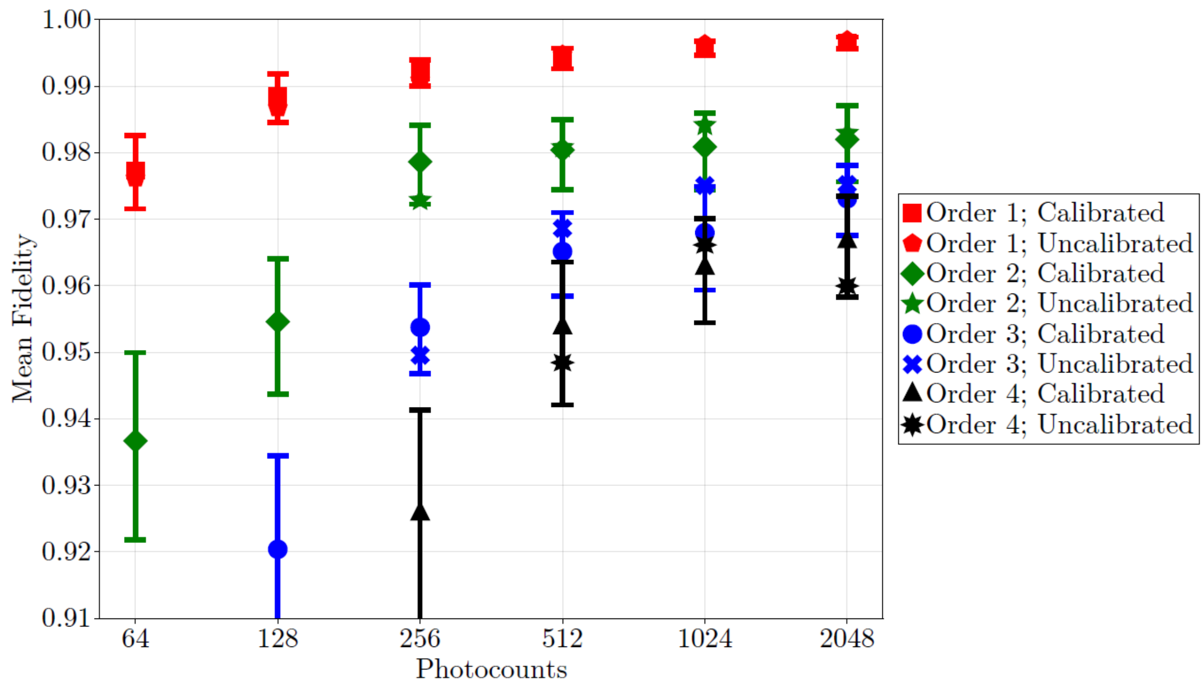
$\{\Pi(R_j), \Pi_\theta(R_j)\}$ is informationally complete for a suitable θ

Experimental results in the photocount regime



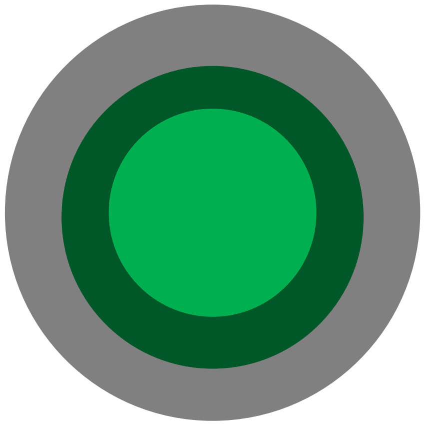


d = 4



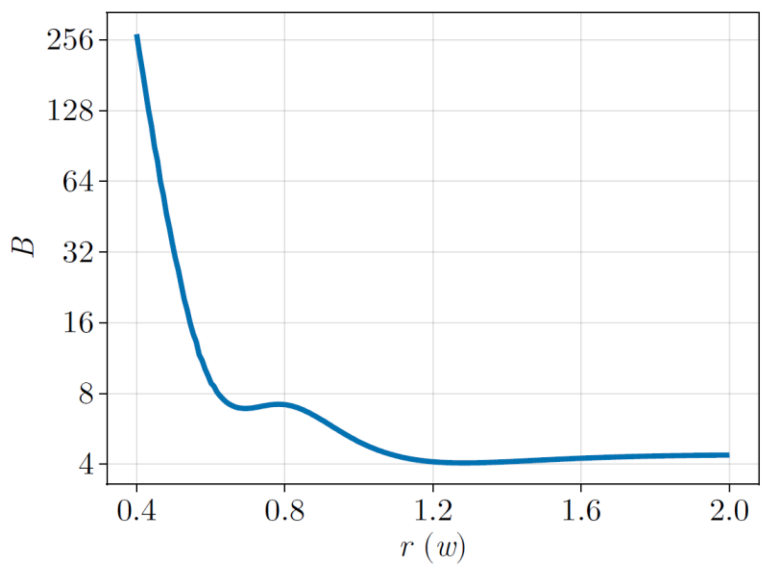
Tomography with partial information

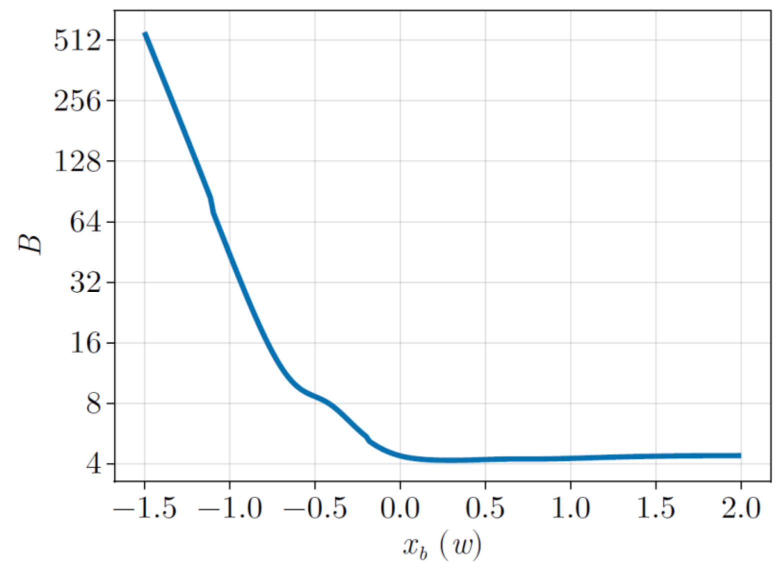
IRIS



BLADE

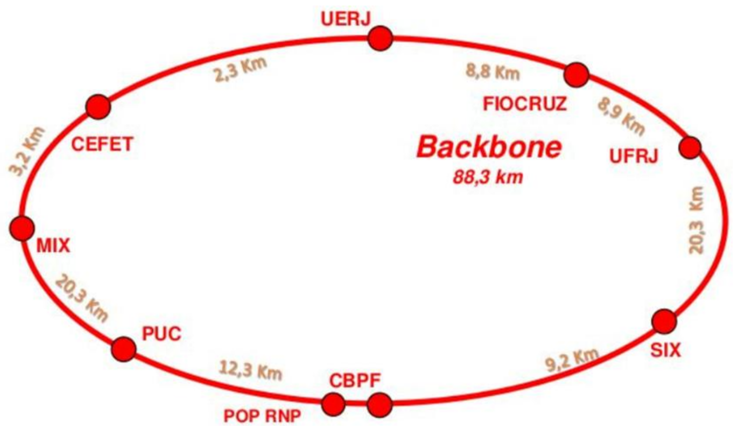
$$\left\|\rho_{\exp} - \rho_{theo}\right\|^2 \geq B \equiv Tr\left(I^{-1}\right)$$

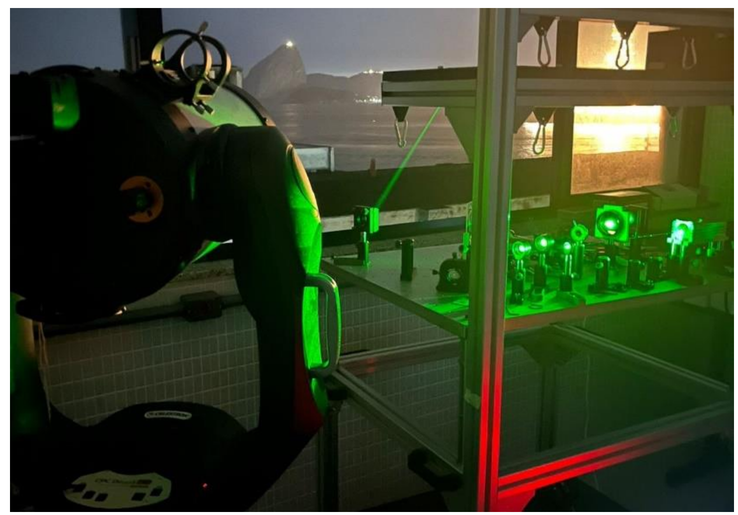




The Rio Quantum Network

Arquitecture





Objectives

- Establishment of a QKD network connecting 5 institutions
- Development of new devices for quantum communication
- Promote scientific exchange
- Testbed for new technologies in q-comm.
- Integration with other national and international networks

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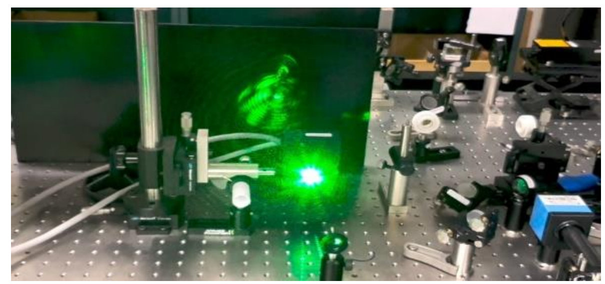
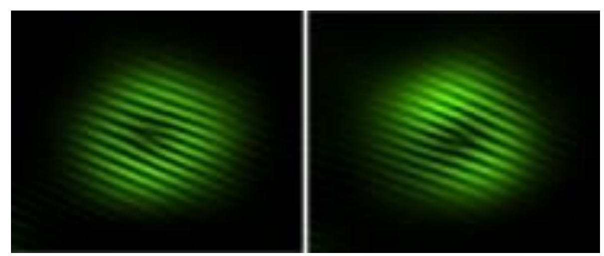
Conclusions

- Quantum tomography in high dimension is a difficult task
- Intensity measurement is an easy task and provide a lot of information
- Reconstruction of structured light state in high dimension with position POVM
- Problem: informational completeness
- Solution: astigmatic transformation





THANK YOU!!!