Broadband variational measurement using strongly nondegenerate dichromatic optical pump

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2 Broadband variational measurement

3 Features of this scheme

4 Splitting

Standard Quantum Limit (SQL)

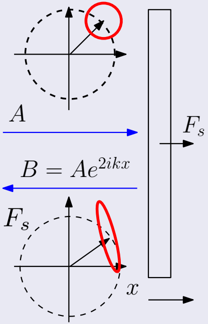
Free mass

$$-m\Omega^2 x = F_s + 2\sqrt{2}\hbar k A a_a$$

Quadratures of the reflected light:

$$b_a = a_a,$$

$$b_\phi = a_\phi + 2\sqrt{2}Akx = a_\phi - \frac{8\hbar k^2 A^2}{m\Omega^2} a_a - \frac{2\sqrt{2}\hbar k A}{m\Omega^2} F_s$$



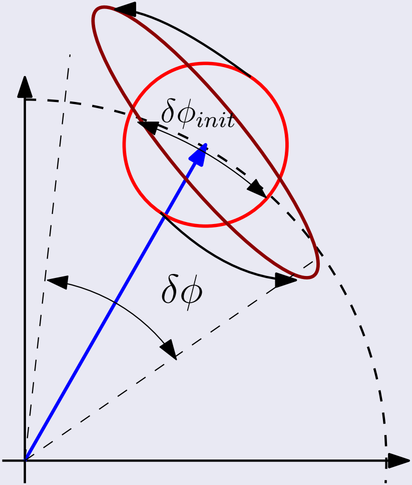
Measurement of phase quadrature:

$$\mathcal{K} = \frac{8\hbar k^2 A^2}{m\Omega^2}, \quad f_s = \frac{F_s}{F_{SQL}}, \quad F_{SQL} = \sqrt{2\hbar m\Omega^2},$$

$$b_\phi = \sqrt{2\mathcal{K}}\left(\frac{a_\phi}{\sqrt{2\mathcal{K}}} - \sqrt{\frac{\mathcal{K}}{2}} a_a + f_s\right), \quad S_f = \frac{1}{2\mathcal{K}} + \frac{\mathcal{K}}{2} \geq 1.$$

SQL

Illustration



Amplitude fluctuations does not change.

Amplitude fluctuations transforms into phase ones: $\delta\phi > \delta\phi_{\text{init}}$

The more incident power the more strong transform — larger $\delta\phi$

There is optimum power of incident light — Standard Quantum Limit.

Conventional variatonal measurement (CVM): free mass

Quadratures of the reflected light:

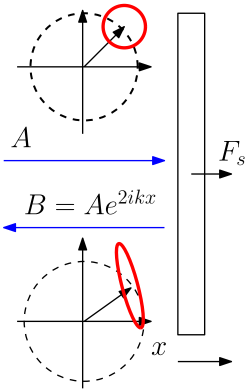
$$b_a = a_a,$$

$$b_\phi = a_\phi - \mathcal{K} a_a - \sqrt{2\mathcal{K}} f_s$$

Conventional variational measurement:

$$b_\theta = b_a \cos\theta + b_\phi \sin\theta =$$

$$= a_a \left(\cos\theta - \mathcal{K}\sin\theta\right) + \left(a_\phi - \sqrt{2\mathcal{K}} f_s\right)\sin\theta$$

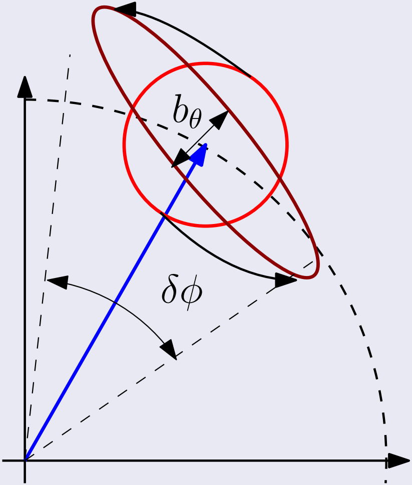


Back action can be compensated on one chosen frequency ($\mathcal{K} = \mathcal{K}(\Omega)$).
CVM: SQL can be surpassed, but in narrow frequency band:

$$S_f = \frac{1}{2\mathcal{K}} + \left(\frac{\cot\theta}{\sqrt{2\mathcal{K}}} - \sqrt{\frac{\mathcal{K}}{2}}\right)^2 < 1.$$

Conventional variatonal measurement (CVM): free mass

Illustration



CVM: to measure quadrature b_θ (not b_ϕ). Back action evasion (!).

Drawbacks:

a) signal became smaller (vulnerable to thermal and technical noise) ;

b) back action evasion takes place only in some bandwidth, the larger pump power, the smaller bandwidth (reason: $\mathcal{K}$ depends on Ω).

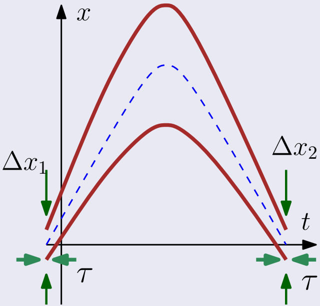
Surpassing of SQL for mechanical oscillator

Stroboscopic measurement

Variational measurement is good for free test mass, but not for mechanical oscillator.

Stroboscopic (instantaneous) 2 measurements of position separated by time $2/T_m$[a].

Short measurement ⇒ large pump. Difficult to realize. . .



[a] V.B. Braginsky, Y.I. Vorontsov, and K.S. Thorne, Science **209**, 547 (1980)

Stroboscopic and Quadrature measurement

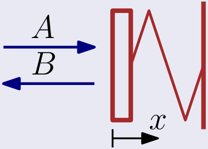
Quadrature measurement

Stroboscopic measurement is equivalent to quadrature measurement:

$$x(t) = d_a \cos\omega_m t + d_\phi \sin\omega_m t,$$

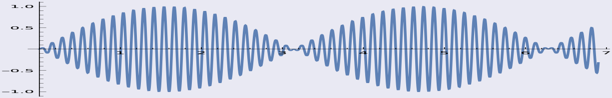
$$x(t) \times \cos\omega_m t \simeq d_a/2,$$

$$B \simeq A(1+2ikx), \;\Rightarrow\; A = A_0 \cos\omega_m t e^{i\omega_0 t}$$



Double pump

$$A* = A_0 \cos\omega_m t e^{i\omega_0 t} = \frac{A_0}{2}\left(e^{i(\omega_0+\omega_m)t} + e^{i(\omega_0-\omega_m)t}\right)$$



Double pump

Idea — to measure not coordinate but one of quadratures of mechanical oscillator, using double pump[a]:

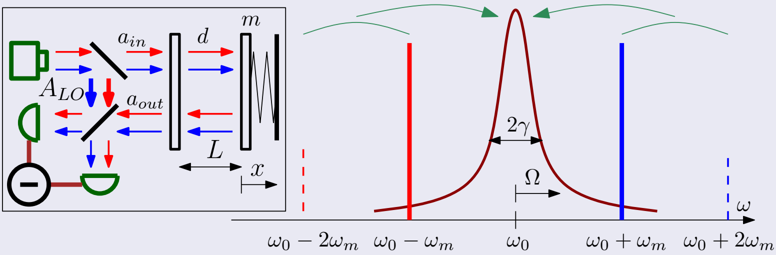


Figure: Measurement of mechanical quadratures. Sidebands $\omega_0 \pm 2\omega_m$ — residual back action.

[a]V.B. Braginsky, Y.I. Vorontsov, and K.S. Thorne, Science **209**, 547 (1980). A. Clerk, F. Marquardt, and K. Jacobs, New Journal of Physics **10**, 095010 (2008).

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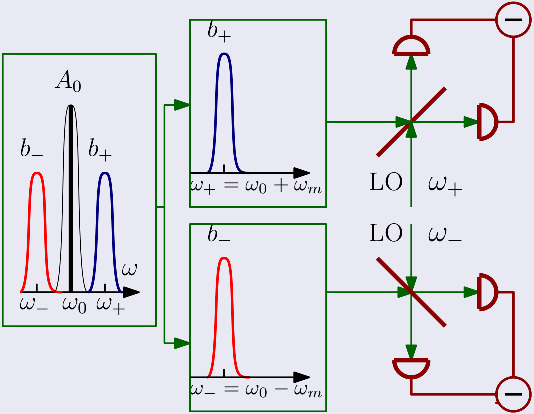
The model with three modes. TWO outputs (!)

Idea: separate detection of sidebands



Optical resonator is pumped on the middle frequency, and two output sidebands $\omega_\pm$ are measured independently.[a]

[a] S. Vyatchanin, A. Nazmiev, A. Matsko, Physical Review A, **106**, 053711 (2022)

Quadratures. TWO outputs (!)

Output fields b_+ and b_- can be separated and
amplitude quadratures b_{+a} and b_{-a} can be measured independently.

Combinations of quadratures

$$\alpha_{\pm a} = \frac{a_{+a} \pm a_{-a}}{\sqrt{2}}, \qquad \alpha_{\pm \phi} = \frac{a_{+\phi} \pm a_{-\phi}}{\sqrt{2}},$$
$$\beta_{\pm a} = \frac{b_{+a} \pm b_{-a}}{\sqrt{2}}, \qquad \beta_{\pm \phi} = \frac{b_{+\phi} \pm b_{-\phi}}{\sqrt{2}}.$$

Sum and difference of output amplitude quadratures:

$$\beta_{+a} = \underbrace{\frac{\gamma + i\Omega}{\gamma - i\Omega}\alpha_{+a}}_{\text{shot noise}}, \qquad \beta_{-a} = \underbrace{\frac{\gamma + i\Omega}{\gamma - i\Omega}\alpha_{-a}}_{\text{shot noise}} + \frac{2\sqrt{\gamma}|\eta C|}{\gamma - i\Omega} d_\phi,$$

$$(\gamma_m - i\Omega) d_\phi = -\underbrace{\frac{2\sqrt{\gamma}|\eta C|}{(\gamma - i\Omega)}\alpha_{+a}}_{\text{back-action}} + \underbrace{\sqrt{2\gamma_M} q_\phi + f_a}_{\text{thermal noise and signal}} .$$

Measurement

Output sum and difference

$$\beta_{+a} = \frac{\gamma + i\Omega}{\gamma - i\Omega}\alpha_{+a},$$

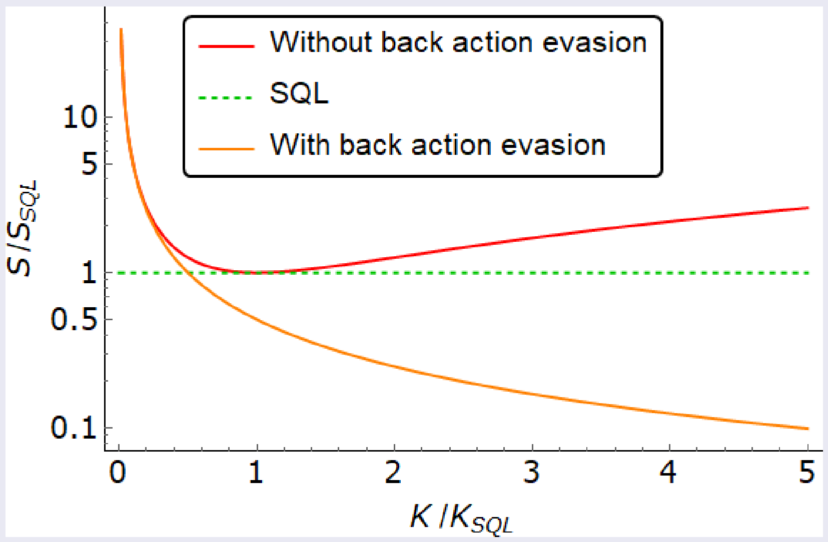
$$\beta_{-a} = \frac{\gamma + i\Omega}{\gamma - i\Omega}\alpha_{-a} - \frac{4\gamma|\eta C|^2}{(\gamma - i\Omega)^2}\frac{\alpha_{+a}}{\gamma_m - i\Omega} + \frac{2\sqrt{\gamma}|\eta C|}{\gamma - i\Omega}\frac{\sqrt{2\gamma_M}q_\phi + f_a}{\gamma_m - i\Omega}.$$

2 outputs: measured combinations

We measure sum β_{+a} and difference β_{-a}.
Post-processing: subtraction of back action term **completely (!)**.

- Continuous measurement.
- Presence of back action.
- Measurement in two channels.
- Complete back action evasion after post-processing.

Plots


Without back action evasion
SQL
With back action evasion
10
5
1
0.5
0.1
S/S_{SQL}
0
1
2
3
4
5
K/K_{SQL}

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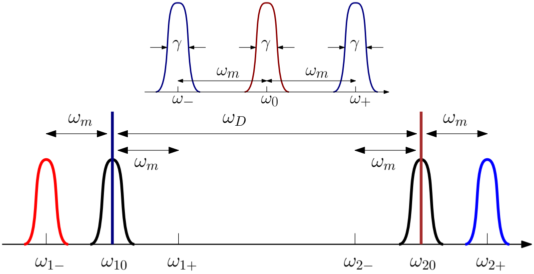
Splitting

How to select optical modes?

The main difficulty to separate modes $\omega_+,\ \omega_-$ — relatively small mechanical frequency (1 – 100 MHz, too small in optics scale).
Proposition: "dividual" ("spased") broadband variational measurement: two nearly degenerate optical mode doublets detuned from each other by several tens of GHz.



No problem to make separate balanced homodyne detection of modes $\omega_{1-},\ \omega_{2+}$

Splitting (cont.)

Model

Two equal pumps on ω_{10} ana ω_{20}.
Output of modes $\omega_{1-},\ \omega_{2+}$ are detected separately.
Post-processing.

$$\omega_{1-} = \omega_{10} - \omega_m, \quad \omega_{2+} = \omega_{20} + \omega_m, \tag{1}$$

$$\gamma_m \ll 1/\tau \ll \gamma \ll \omega_m. \tag{2}$$

Thermal noise

Thermal (Nyquist) noise in mechanical oscillator. It can be considerably reduced if measurement time τ is much smaller than the mechanical ring down time γ_m^{-1}.

Payment

Presence of vacuum fluctuations around frequencies $\omega_{1+},\ \omega_{2-}$. They produce not compensated back action.

Details of calculations

Basic equations

$$\dot{\hat{c}}_{1-} + \gamma \hat{c}_{1-} = \eta C_{10} \hat{d}^\dagger + \sqrt{2\gamma}\hat{a}_{1-}$$

$$\dot{\hat{c}}_{2+} + \gamma \hat{c}_{2+} = -\eta C_{02} \hat{d} + \sqrt{2\gamma}\hat{a}_{2+},$$

$$\dot{\hat{d}} + \gamma_m \hat{d} = \eta \Big(C_{10} \hat{c}_{1-}^\dagger + C_{20}^* \hat{c}_{2+} + + C_{10}^* \hat{c}_{1+} + C_{20} \hat{c}_{2-}^\dagger \Big) + \sqrt{2\gamma_m}\, \hat{q} + f_s.$$

Fluctuation operators

$$\left[\hat{a}_{i,j\pm}(t), \hat{a}_{i,j\pm}^\dagger(t') \right] = \delta_{ij}\, \delta(t-t'),$$

$$\left[\hat{q}(t), \hat{q}^\dagger(t') \right] = \delta(t-t'),$$

$$\left\langle \hat{q}(t) \hat{q}^\dagger(t') \right\rangle = (2n_T + 1)\, \delta(t-t'),$$

$$n_T = \left(e^{\hbar\omega_m/\kappa_B T} - 1 \right)^{-1}.$$

Details of calculations (cont.)

Parasitic modes

$$\dot{\hat{c}}_{1+} + (\gamma - i\omega_m)\hat{c}_{1+} = -\eta C_{10}\hat{d} + \sqrt{2\gamma}\hat{a}_{1+}$$
$$\dot{\hat{c}}_{2-} + (\gamma + i\omega_m)\hat{c}_{2-} = \eta C_{02}\hat{d}^\dagger + \sqrt{2\gamma}\hat{a}_{2-}.$$

Input – output relations and output amplitude quadratures

$$\hat{b}_{1-} = -\hat{a}_{1-} + \sqrt{2\gamma}\,\hat{c}_{1-},$$
$$\hat{b}_{2+} = -\hat{a}_{2+} + \sqrt{2\gamma}\,\hat{c}_{2+}.$$

$$b_{a1-} = \frac{\sqrt{2\gamma}\,\eta C_0 d_a}{\gamma - i\Omega} + \frac{\gamma + i\Omega}{\gamma - i\Omega}\,a_{a1-},$$
$$b_{a2+} = \frac{-\sqrt{2\gamma}\,\eta C_0 d_a}{\gamma - i\Omega} + \frac{\gamma + i\Omega}{\gamma - i\Omega}\,a_{a2+}.$$

Post-processing analysis

Sum and difference of amplitude quadratures

$$\beta_{a\pm} = \frac{b_{a1-} \pm b_{a2+}}{\sqrt{2}}, \quad \alpha_{a\pm} = \frac{a_{a1-} \pm a_{a2+}}{\sqrt{2}}\,,$$

$$\beta_{a+} = \xi\,\alpha_{a+}, \quad \xi \equiv \frac{\gamma + i\Omega}{\gamma - i\Omega},$$

$$\beta_{a-} = \frac{-2\sqrt{\gamma}\,\eta C_0 d_a}{\gamma - i\Omega} + \xi\,\alpha_{a-},$$

$$d_a = \frac{2\sqrt{\gamma}\,\eta C_0\left(\alpha_{a+} - \frac{\gamma - i\Omega}{\omega_m}\alpha_{\phi-}\right)}{(\gamma_m - i\Omega)(\gamma - i\Omega)} + \frac{\sqrt{2\gamma_m}\,q_a + f_{sa}}{\gamma_m - i\Omega}.$$

Measurement of β_{a+} — main back action ($\sim \alpha_{a+}$) (can be subtracted from β_{a-} in post-processing). The blue back action term ($\sim \alpha_{\phi-}$) defines limit sensitivity.

Post-processing analysis (cont.)

After subtraction of main back action

$$\tilde{\beta}_{a-} = \frac{\sqrt{\xi\mathcal{K}}}{\gamma_m - i\Omega}\Bigg((\gamma_m - i\Omega)\sqrt{\frac{\xi}{\mathcal{K}}}\,\alpha_{a-} + \\ + \sqrt{\xi\mathcal{K}}\,\frac{\gamma - i\Omega}{\omega_m}\,\alpha_{\phi-} - \sqrt{2\gamma_m}\,q_a + f_{sa}\Bigg), \quad \mathcal{K} \equiv \frac{4\gamma\,\eta^2 C_0^2}{\gamma^2 + \Omega^2}\,.$$

Spectral density

Single-sided spectral density of the noise reevaluated for the normalized force quadrature f_a :

$$S_{fa} = S_T + S_{qu}, \quad S_T = 2\gamma_m\left(n_T + \frac{1}{2}\right),$$

$$S_{qu} = \frac{|\gamma_m - i\Omega|^2}{\mathcal{K}} + \frac{\mathcal{K}|\gamma - i\Omega|^2}{\omega_m^2} \simeq \frac{\Omega^2}{\mathcal{K}} + \mathcal{K}\,\frac{\gamma^2}{\omega_m^2}$$

Plot

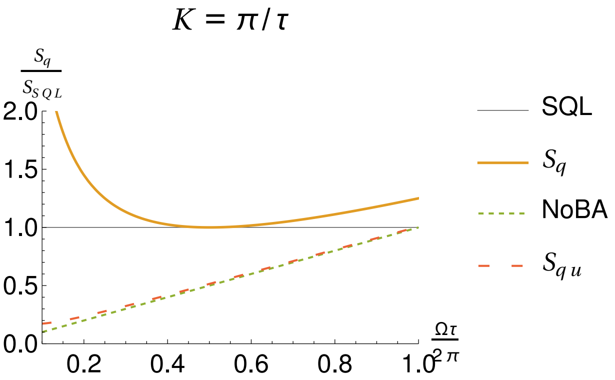


Figure: Power spectral density of the measurement noise normalized to SQL in several cases. The curve marked by S_q relates to the usual SQL. The curve marked with "NoBA" corresponds to the case of the complete back action evasion . The curve marked by S_{qu} illustrates the partial back action evasion characterized with the coefficient $(\gamma/\omega_m)^2 \simeq 0.03$. Here time τ is a duration of the signal resonance force envelope.

Conclusion

1. We have investigated a broadband multidimensional variation measurement of a force acting on a mechanical oscillator using a strongly nondegenerate dichromatic optical pump.
2. We found that the scheme supports the broadband back action evading measurement and also allows straightforward experimental implementation.
3. The back action compensation is not complete in the presence of off-resonant modulation sidebands, they restrict the value of back action correlation in the two measurement channels and, hence, sensitivity.

MANY THANKS FOR ATTENTION!