

## Thermal coordinates and black hole thermodynamics

D. O. Stepanenko<sup>1</sup>

In black hole thermodynamics, two fundamental questions arise: the black hole explosion problem and the violation of the third law of thermodynamics.

The black hole explosion problem, as discussed in [3], arises from the expression for Hawking temperature,  $T = \frac{1}{8\pi M}$ , which diverges as the mass  $M \rightarrow 0$ . A proposed solution for Schwarzschild black holes was introduced in [1], involving a modification of Kruskal coordinates by introducing thermal coordinates, represented as:

$$T = \frac{1}{\mathcal{B}(M)} = \frac{1}{2\pi(4M + b)} \quad (1)$$

As  $M$  approaches a small value, this formulation prevents the temperature from diverging, instead allowing it to settle at a constant value.

However, this method does not address the violation of the third law of thermodynamics [2], which states that entropy should approach zero as temperature approaches zero. Indeed, the entropy of a black hole depends on temperature as:

$$S = \frac{1}{16\pi T^2} \quad (2)$$

Thus, as  $T \rightarrow 0$ , a violation of the third law of thermodynamics occurs, as formulated by Planck. In this talk, we propose a generalization of thermal coordinates [4] that addresses both the violation of the third law and the black hole explosion problem.

## References

1. *Aref'eva I., Volovich I.* Quantum Explosions of Black Holes and Thermal Coordinates // Symmetry. — 2022. — Dec. — Vol. 14, no. 11. — P. 2298. — ISSN 2073-8994. — DOI: 10.3390/sym14112298.

---

<sup>1</sup>Steklov Mathematical Institute RAS, Moscow Institute of Physics and Technology.  
Email: dstepanenko@mi-ras.ru

2. *Aref'eva I., Volovich I.* Violation of the third law of thermodynamics by black holes, Riemann zeta function and Bose gas in negative dimensions // The European Physical Journal Plus. — 2024. — Mar. — Vol. 139, no. 3. — ISSN 2190-5444. — DOI: 10.1140/epjp/s13360-024-05049-7.
3. *Hawking S. W.* Black hole explosions? // Nature. — 1974. — Mar. — Vol. 248, no. 5443. — P. 30–31. — ISSN 1476-4687. — DOI: 10.1038/248030a0.
4. *Stepanenko D., Volovich I.* Thermal coordinates and black hole thermodynamics, [In preparation].