Ergodicity of invariant measures on a Hilbert space and pecularities of state diffusion

V. Zh. Sakbaev[1]

We study a finitely additive nonnegative measure λ on a real separable Hilbert space E that are invariant with respect to the group of shifts on any vector of this space.

The Hilbert space $H = L_2(E, \lambda, C)$ of function that are squre integrable with respect to an invariant measure is intoduced. We show that the representation π of the Abelian group E in the space H is not strongly continuous and we find the subgroup L such that the mapping $\pi : L \to B(H)$ is continuous in the strong operator topology.

Let G be a group of mapping of a space E into itself. Let R be a G-invariant ring of subsets of a space E.

Definition. A G-invariant measure $\mu : R \to [0, +\infty)$ is called

1. *ring-decomposible* with respect to the group G if there are two G-invariant subrings r_1, r_2 of the ring R satysfying conditions a), b) such that $\mu(A) = 0 \ \forall \ A \in r_1 \bigcap r_2$; where
   a) $\mu|_{r_1} \neq 0, \ \mu|_{r_2} \neq 0$,
   b) the ring R is completion with respect to the measure μ of the ring which is generated by the collection of sets $r_1 \bigcup r_2$;
2. *ring-ergodic* with respect to the group G if for any two G-invariant subrings r_1, r_2 of the ring R the conditions a), b) imply that there is $A \in r_1 \bigcap r_2$ such that $\mu(A) > 0$.

The decomposition of a G-invariant measure μ to the sum of ring-ergodic with respect to the group G mutually singular measures is called *ring-ergodic with respect to the group G decomposition of the G-invariant measure μ*.

We obtain the ring-ergodic with respect to the group L decompositions $\lambda = \oplus_{z \in E/L} \lambda_z$ and the orthogonal decomposition $H = \oplus_{z \in E/L} H_z$ where $H_z = L_2(E, \lambda_z, C)$.

[1]Keldysh Institute of Applied Mathematics RAS, Ufa Mathematical Institute UFRC RAS.
Email: fumi2003@mail.ru

We prove, that the representation **U** of the convolutional semigroup of Gaussian measures on the space E in the space H is strongly continuous if and only if subspaces $H_z,\ z \in E/L$, are invariant with respect to the semigroup **U**.