Equivalence of Ensembles for a Zero–Range Process on Sparse Configuration–Model Graphs. E.Yu. Kalimulina (MSU, IITP RAS, Moscow, Russia)

We study the zero–range process (ZRP) on a sparse random network generated by the configuration model. For each N, the graph G_N has N vertices with i.i.d. degrees $\{k_i^{(N)}\}_{i=1}^N$ satisfying $\max_i k_i^{(N)} \leq C\sqrt{N}$ for some fixed $C > 0$; the empirical degree distribution $\Pi_N(k) = N^{-1}\sum_{i=1}^N \mathbf{1}_{\{k_i^{(N)}=k\}}$ converges pointwise to a limit $\Pi(k)$ as $N \to \infty$. Particles hop from x with occupation number m at rate $u(m)$, where the single–site weights $p(m) = \prod_{n=1}^m u(n)^{-1}$ decay exponentially, $p(m) \leq a\,\mathrm{e}^{-bm}$. Let M_N be the total number of particles with density $\rho = M_N/N$, assumed to converge to $\rho < \rho_c$, the critical density of the corresponding grand–canonical ensemble.

Define the *effective* weights

$$\widehat{p}_N(m) = p(m)\sum_{k\geq 1}\Pi_N(k)\,k^m, \qquad \widehat{p}(m) = \lim_{N\to\infty}\widehat{p}_N(m),$$

and set $Z(z) = \sum_{m\geq 0}\widehat{p}(m)z^m$. The fugacity $z = z(\rho) \in (0,1)$ is the unique solution of

$$\rho = \frac{\sum_{m\geq 0} m\widehat{p}(m)z^m}{Z(z)}.$$

Theorem 1. *For any fixed $m \in \mathbb{N}_0$, the canonical measure $\mathbb{P}_N$ of the ZRP on G_N satisfies*

$$\lim_{N\to\infty}\mathbb{P}_N\big(\eta(x) = m\big) = \frac{\widehat{p}(m)z^m}{Z(z)} =: \nu_\rho(m), \qquad \textit{and} \qquad \|\mathbb{P}_N - \nu_\rho^{\otimes N}\|_{\mathrm{TV}} \leq \mathrm{e}^{-cN},$$

for some $c > 0$ independent of N, and $\lim_{N\to\infty}\frac{1}{N}\log Z_N(M_N) = \log Z(z) - z\,\rho$.