Confidence sets in portfolio control

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One-dimensional data. Clustering.Confidence interval calculation.

Problem.

$$\min_{m,\delta} \sum_{i=1}^{N} [(\rho_i - m_1)^2 \delta_i + (\rho_i - m_2)^2 (1 - \delta_i)]$$

Sample $V = \{\rho_1, ..., \rho_N\}$ is sorted in ascending order.

Algorithm.

1.Define $V_1 = \{\rho_1\}, V_2 = \{\rho_2, ..., \rho_N\}, k = 1.$

2. Calculate $m_1 = \frac{1}{k} \sum_{i=1}^{k} \rho_i, m_2 = \frac{1}{N-k} \sum_{i=k+1}^{N} \rho_i, k = k + 1.$

3. Calculate the next partition $W = \begin{cases} V_1 + \{\rho_{k+1}\}, d(\rho_{k+1}, m_1) < d(\rho_{k+1}, m_2) \\ V_1, \text{otherwise} \end{cases}$

$V_2 = V \backslash W.$

4. If $W = V_1$, then stop; else $V_1 = W$, go to 2.

One-dimensional data. Clustering.Confidence interval calculation.

Dichotomous clustering algorithm.

1. Initialization. Let $V = V_1^1 \bigcup V_2^1, k = 1$.
2. Iteration. Let $V = V_1^k \bigcup V_2^k \bigcup \ldots \bigcup V_{k+1}^k$.

For each cluster calculate $d_j^k = \sum\limits_{\rho \in V_j^k} (\rho - m_j^k)^2$.

Define cluster with maximum $d_{j^*}^k = \max\{d_1^k, d_2^k, ..., d_{k+1}^k\}$.
The selected cluster is splitted into two clusters, $k = k + 1$.

One-dimensional data. Clustering.Confidence interval calculation.

Calculation a minimal length interval, that contains a given number L of sample elements.
Sample $V = \{\rho_1, ..., \rho_N\}$ is sorted in ascending order.

Algorithm.

1.Define $i = 1, j = N$.

2. Calculate $i = \begin{cases} i+1, & \rho_{i+1} - \rho_i > \rho_j - \rho_{j-1} \\ i, & \rho_{i+1} - \rho_i \leq \rho_j - \rho_{j-1} \end{cases}$,

$j = \begin{cases} j, & \rho_{i+1} - \rho_i > \rho_j - \rho_{j-1} \\ j-1, & \rho_{i+1} - \rho_i \leq \rho_j - \rho_{j-1} \end{cases}$.

3. If $k > N - L$, then $a = \rho_i, b = \rho_j$. Stop.

Sellers problem. Call option, $f = f(S_N)$.
Formulation.

$$X_0(x) = \min_{\gamma_1^N} y, \frac{y}{B_0} + \frac{1}{1+r}\sum_{i=1}^{N}\gamma_i \frac{S_{i-1}}{B_{i-1}}(\rho_i - r) \geq \frac{f\left(x\prod_{i=1}^{N}(1+\rho_i)\right)}{B_N},$$

$\forall \rho_i \in [a_1, a_2] \cup [a_3, a_4]$, $\Delta S_i = S_{i-1}\rho_i, \Delta B_i = B_{i-1}r, x = S_0,$
$\gamma_r^s = \{\gamma_r, \gamma_{r+1}, ..., \gamma_s\}.$
Solution.

$$X_k(x) = \frac{1}{(1+r)^{N-k}}\sum_{j=0}^{N-k} C_{N-k}^{j} f(x(1+a_4)^j(1+a_1)^{N-k-j})(p^*)^j(q^*)^{N-k-j},$$

$$p^* = \frac{r-a_1}{a_4-a_1}, q^* = 1-p^*,$$

$$\gamma_{k+1}(x) = \frac{X_{k+1}(x(1+a_4)) - X_{k+1}(x(1+a_1))}{x(a_4-a_1)}, x = S_k.$$

One-dimensional data. Hedging in (B,S)-market.

Buyers problem. Call option, $f = f(S_N)$.
Formulation.

$$X_0(x) = \max_{\gamma_1^N} y, \frac{y}{B_0} + \frac{1}{1+r}\sum_{i=1}^{N} \gamma_i \frac{S_{i-1}}{B_{i-1}}(\rho_i - r) \le \frac{f\left(x\prod_{i=1}^{N}(1+\rho_i)\right)}{B_N},$$

$\forall \rho_i \in [a_1, a_2] \cup [a_3, a_4]$, $\Delta S_i = S_{i-1}\rho_i, \Delta B_i = B_{i-1}r, x = S_0,$
$\gamma_r^s = \{\gamma_r, \gamma_{r+1}, \ldots, \gamma_s\}$.

Solution. Let $r \in (a_2, a_3)$.

$$X_k(x) = \frac{1}{(1+r)^{N-k}}\sum_{j=0}^{N-k} C_{N-k}^{j} f(x(1+a_3)^j(1+a_2)^{N-k-j})(p^*)^j(q^*)^{N-k-j},$$

$$p^* = \frac{r-a_2}{a_3-a_2}, q^* = 1-p^*,$$

$$\gamma_{k+1}(x) = \frac{X_{k+1}(x(1+a_3)) - X_{k+1}(x(1+a_2))}{x(a_3-a_2)}, x = S_k.$$

Solution. Let $r \in (a_1, a_2) \cup (a_3, a_4)$.

$$X_k(x) = \frac{1}{(1+r)^{N-k}} f(x(1+r)^{N-k}),$$

$\gamma_{k+1}(x)$ - arbitrary, $x = S_0(1+r)^k$.

One-dimensional data. Hedging in (B,S)-market.

Sellers problem. Option with aftereffect, $f = f\left(S_N, \sum_{i=0}^{N} S_i\right)$.

Formulation.

$$X_0(u,v) = \min_{\gamma_1^N} y, \frac{y}{B_0} + \frac{1}{1+r} \sum_{i=1}^{N} \gamma_i \frac{S_{i-1}}{B_{i-1}} (\rho_i - r) \geq \frac{f\left(u \prod_{i=1}^{N}(1+\rho_i), v + \sum_{i=1}^{N} S_i\right)}{B_N},$$

$\forall \rho_i \in [a_1, a_2] \cup [a_3, a_4]$, $\Delta S_i = S_{i-1}\rho_i, \Delta B_i = B_{i-1} r, u = S_0, v = S_0,$
$\gamma_r^s = \{\gamma_r, \gamma_{r+1}, \ldots, \gamma_s\}$.

Solution.

$X_k(u,v) = \frac{1}{1+r}(q^* X_{k+1}(u(1+a_1), v + u(1+a_1)) + p^* X_{k+1}(u(1+a_4), v + u(1+a_4)))$,

$X_N(u,v) = f_N(u,v), p^* = \frac{r - a_1}{a_4 - a_1}, q^* = 1 - p^*,$

$$\gamma_{k+1}(u,v) = \frac{X_{k+1}(u(1+a_4), v + u(1+a_4)) - X_{k+1}(u(1+a_1), v + u(1+a_1))}{u(a_4 - a_1)},$$

$$u = S_k, v = \sum_{i=0}^{k} S_i.$$

One-dimensional data. Hedging in (B,S)-market.

Buyers problem. **Option with aftereffect**, $f = f\left(S_N, \sum_{i=0}^{N} S_i\right)$.

Formulation.

$$X_0(u,v) = \max_{\gamma_1^N} y, \frac{y}{B_0} + \frac{1}{1+r}\sum_{i=1}^{N}\gamma_i \frac{S_{i-1}}{B_{i-1}}(\rho_i - r) \leq \frac{f\left(u\prod_{i=1}^{N}(1+\rho_i), v + \sum_{i=1}^{N} S_i\right)}{B_N},$$

$\forall \rho_i \in [a_1, a_2] \cup [a_3, a_4]$, $\Delta S_i = S_{i-1}\rho_i, \Delta B_i = B_{i-1}r, u = S_0, v = S_0,$

$\gamma_r^s = \{\gamma_r, \gamma_{r+1}, \ldots, \gamma_s\}$.

Solution. Let $r \in (a_2, a_3)$.

$X_k(u,v) = \frac{1}{1+r}(q^* X_{k+1}(u(1+a_2), v + u(1+a_2)) + p^* X_{k+1}(u(1+a_3), v +$
$u(1+a_3)))$,

$X_N(u,v) = f_N(u,v), p^* = \frac{r-a_2}{a_3-a_2}, q^* = 1-p^*,$

$$\gamma_{k+1}(u,v) = \frac{X_{k+1}(u(1+a_3), v+u(1+a_3)) - X_{k+1}(u(1+a_2), v+u(1+a_2))}{u(a_3-a_2)},$$

$u = S_k, v = \sum_{i=0}^{k} S_i$.

Solution. Let $r \in (a_1, a_2) \cup (a_3, a_4)$.

$$X_k(u,v) = \frac{1}{(1+r)^{N-k}} f\left(u(1+r)^{N-k}, v + S_0 \frac{(1+r)^{N+1} - (1+r)^{k+1}}{r}\right)$$

$$u = S_k, v = \sum_{i=0}^{k} S_i, S_i = S_{i-1}(1+r),$$

$\gamma_{k+1}(u,v)$ - arbitrary.

Multidimensional data. Clustering. Confidence ellipsoid calculation.

Problem.
$$\min_{m,\delta} \sum_{i=1}^{N} \sum_{j=1}^{n} (\rho_i - m_j)^2 \delta_{i,j}, \delta_{i,j} \in [0,1], \sum_{j=1}^{n} \delta_{i,j} = 1.$$

Algorithm.
Let $V = \{\rho_1, ..., \rho_N\}$ be the sample, C be the sample covariance matrix. Define ε.
Define eigenvector l:$\|l\| = 1$, which corresponds to minimal eigenvalue of matrix C.
1.Calculate $U = \{u_1, ..., u_N\}, u_i = (l, \rho_i), k = 1$.
2.Divide sample U into two clusters U_1 and U_2. Define $\delta_i = \begin{cases} 1, & u_i \in U_1 \\ 0, & u_i \in U_2 \end{cases}$.

Multidimensional data. Clustering. Confidence ellipsoid calculation.

3. If $k > 1$ then $M_1 = m_1, M_2 = m_2$.

Calculate $m_1 = \dfrac{\sum\limits_{i=1}^{N} \delta_i \rho_i}{\sum\limits_{i=1}^{N} \delta_i}, m_2 = \dfrac{\sum\limits_{i=1}^{N} (1-\delta_i)\rho_i}{\sum\limits_{i=1}^{N} (1-\delta_i)}$,

$$C_{\text{in}} = \sum_{i=1}^{N} \delta_i (\rho_i - m_1)(\rho_i - m_1)^T + \sum_{i=1}^{N} (1-\delta_i)(\rho_i - m_2)(\rho_i - m_2)^T.$$

4. Find vector l, solving the problem: $\min\limits_{\|l\|=1} \dfrac{(C_{\text{in}} l, l)}{(m_2 - m_1, l)^2}$.

5. If $k > 1$ and $\|m_1 - M_1\| < \varepsilon$ and $\|m_2 - M_2\| < \varepsilon$ then stop; else $k = k+1$, go to 1.

Multidimensional data. Clustering. Confidence ellipsoid calculation.

Let
$\sum_i \lambda_i = 1, m(\lambda) = \sum_i \lambda_i \rho_i, C(\lambda) = \sum_i \lambda_i (\rho_i - m(\lambda))(\rho_i - m(\lambda))^T,$
$A(\lambda) = (\Delta(C(\lambda)))^{1/n} C^{-1}(\lambda), \Delta(C(\lambda)) \neq 0.$
Problem:
$\max_\lambda \sum_i \lambda_i (A(\lambda)(\rho_i - m(\lambda)), \rho_i - m(\lambda)), \sum_i \lambda_i = 1, \lambda_i \geq 0.$ (1)

Multidimensional data. Clustering. Confidence ellipsoid calculation.

Algorithm Minimal Ellipsoid (ME).
Define $H_1 = \{\rho_1, ..., \rho_L\}$.
1. Find λ^1 from problem (1).
2. Construct ordered permutation π: $\pi(i) < \pi(j)$, if $(A(\lambda^1)(\rho_{\pi(i)} - m(\lambda^1)), \rho_{\pi(i)} - m(\lambda^1)) < (A(\lambda^1)(\rho_{\pi(j)} - m(\lambda^1)), \rho_{\pi(j)} - m(\lambda^1))$.
3. Form $H_2 = \{\rho_{\pi(i)}, i = 1, ..., L\}$.
4. Find λ^2 from problem (1).
5. If $\Delta(C(\lambda^2)) < \Delta(C(\lambda^1))$, then $\lambda_1 = \lambda_2$,go to 2; else stop.
We obtained subsample (ellipsoid) H_1, center $m(H_1)$ and matrix $C(H_1)$.

Multidimensional data. One-step portfolio.

x - portfolio, ρ - assets return, (x,ρ) - portfolio return,

$D = \bigcup_{i=1}^{p} El_i$, $El_i = \{\rho : (C_i^{-1}(\rho - m_i), \rho - m_i) \leq 1\}$ - confidence set,

$P(\rho \in D) \geq \eta$,

m_i, C_i - center and matrix of ellipsoid El_i.

Define for ellipsoid El_i

$\alpha(i,x) = \min_{\rho \in El_i} (x,\rho)$ - minimal return,

$\beta(i,x) = \max_{\rho \in El_i} (x,\rho)$ - maximal return,

$\gamma(i,x) = \dfrac{\alpha(i,x) + \beta(i,x)}{2}$ - average return,

$\sigma(i,x) = \dfrac{\beta(i,x) - \alpha(i,x)}{2}$ - variance,

$\alpha(i,x) = (x, m_i) - \sqrt{(C_i x, x)}, \beta(i,x) = (x, m_i) + \sqrt{(C_i x, x)}, \gamma(i,x) = (x, a_i)$,

$\sigma(i,x) = \sqrt{(C_i x, x)}$.

Define for portfolio x

$\alpha(x) = \min \alpha(i,x)$ - minimal return,

$\gamma(x) = \min \gamma(i,x)$ - minimal average return,

$\sigma(x) = \max \sigma(i,x)$ - maximal variance.

Multidimensional data. One-step portfolio.

Mean-variance portfolio:

1. $\max_x (x, m), (Cx, x) \le b, (I, x) = 1, m = E\rho, C = E\rho\rho^T - mm^T,$
2. $\max_x \min_i (x, m_i), (C_i x, x) \le b, (I, x) = 1,$
3. $\min_x \max_i (C_i x, x), (x, m_i) \ge b, (I, x) = 1,$
4. $\max_x [\min_i (x, m_i) - \lambda \max_j \sqrt{(C_j x, x)}], (I, x) = 1.$

VaR portfolio:

1. $\max \alpha, P((x, \rho) \ge \alpha) \ge \eta, (I, x) = 1,$
2. $\max \alpha(x), (I, x) = 1 \Leftrightarrow \max_x \min_i ((x, m_i) - \sqrt{(C_i x, x)}), (I, x) = 1.$

Portfolio with minimal risk:

$\min_x \max_i \sqrt{(C_i x, x)}, (I, x) = 1,$

α^* - guaranteed return ,d^* - guaranteed risk (confidence probability η).

Multidimensional data. Dynamic portfolio.

$M = \bigcup_{i=1}^{p} M_i, M_i = [(x, m_i) - \sqrt{(C_i x, x)}, (x, m_i) + \sqrt{(C_i x, x)}]$.

Let $a_1 = \min_i((x, m_i) - \sqrt{(C_i x, x)}), a_4 = \max_i((x, m_i) + \sqrt{(C_i x, x)})$.

Consider two cases:

1. $\forall y \in [a_1, a_4] \Rightarrow y \in M$.In this case we have one interval $[a_1, a_4]$.
2. $\exists y \in [a_1, a_4] : y \notin M$. Let $\bar{M} = [a_1, a_4] \backslash M, a_2 = \inf\{\bar{M}\}, a_3 = \sup\{\bar{M}\}$.

In this case we have two non-intersect intervals: $[a_1, a_2], [a_3, a_4]$.

Multidimensional data. Dynamic portfolio.

The first case.

$S_1 = S_0(1+\rho), \rho \in [a_1, a_4], S_0 = 1,$

$B_1 = B_0(1+r), B_0 = 1,$

$X_{n/N} = \gamma_n S_{n/N} + \beta_n B_{n/N}, S_{n/N} = S_{(n-1)/N}\left(1 + \frac{\rho}{N}\right), B_{n/N} = B_{(n-1)/N}\left(1 + \frac{r}{N}\right),$

$\frac{\rho}{N} \in \left[\frac{a_1}{N}, \frac{a_4}{N}\right], f(S_1) = (\alpha^* - (S_1 - 1))^+,$

$\min X_0, X_1 \geq f(S_1),$

$$C^* = \frac{1}{\left(1 + \frac{r}{N}\right)^N}\left[(\alpha^* + 1)\sum_{j=0}^{j_0} C_N^j (p^*)^j (q^*)^{N-j} - \sum_{j=0}^{j_0} C_N^j (\bar{p})^j (\bar{q})^{N-j}\right],$$

$$p^* = \frac{r - a_1}{a_4 - a_1}, q^* = 1 - p^*, \bar{p} = \left(1 + \frac{a_4}{N}\right)p^*, \bar{q} = \left(1 + \frac{a_1}{N}\right)q^*,$$

$$j_0 = \left\lfloor \log_{i_0} \frac{\alpha^* + 1}{S_0\left(1 + \frac{a_1}{N}\right)^N} \right\rfloor, i_0 = \frac{1 + a_4/N}{1 + a_1/N},$$

$\max X_0, X_1 \leq f(S_1),$

$$C_* = \frac{1}{(1+r)^N}\left(\alpha^* + 1 - \left(1 + \frac{r}{N}\right)^N\right)^+.$$

The second case.

$S_1 = S_0(1+\rho), \rho \in [a_1, a_2] \cup [a_3, a_4], S_0 = 1,$

$B_1 = B_0(1+r), B_0 = 1,$

$X_{n/N} = \gamma_n S_{n/N} + \beta_n B_{n/N}, S_{n/N} = S_{(n-1)/N}\left(1 + \frac{\rho}{N}\right), B_{n/N} = B_{(n-1)/N}\left(1 + \frac{r}{N}\right),$

$\frac{\rho}{N} \in \left[\frac{a_1}{N}, \frac{a_2}{N}\right] \cup \left[\frac{a_3}{N}, \frac{a_4}{N}\right], f(S_1) = (\alpha^* - (S_1 - 1))^+,$

Sellers problem

$\min X_0, X_1 \geq f(S_1),$

$$C^* = \frac{1}{\left(1 + \frac{r}{N}\right)^N}\left[(\alpha^* + 1)\sum_{j=0}^{j_0} C_N^j (p^*)^j (q^*)^{N-j} - \sum_{j=0}^{j_0} C_N^j (\bar{p})^j (\bar{q})^{N-j}\right],$$

$$p^* = \frac{r - a_1}{a_4 - a_1}, q^* = 1 - p^*, \bar{p} = \left(1 + \frac{a_4}{N}\right)p^*, \bar{q} = \left(1 + \frac{a_1}{N}\right)q^*,$$

$$j_0 = \left\lfloor \log_{i_0} \frac{\alpha^* + 1}{S_0\left(1 + \frac{a_1}{N}\right)^N} \right\rfloor, i_0 = \frac{1 + a_4/N}{1 + a_1/N}.$$

Multidimensional data. Dynamic portfolio.

Buyers problem

$\min X_0, X_1 \geq f(S_1)$,

1. $r \in (a_2, a_3)$,

$$C_* = \frac{1}{\left(1+\frac{r}{N}\right)^N}\left[(\alpha^*+1)\sum_{j=0}^{j_0} C_N^j (p^*)^j (q^*)^{N-j} - \sum_{j=0}^{j_0} C_N^j (\bar{p})^j (\bar{q})^{N-j}\right],$$

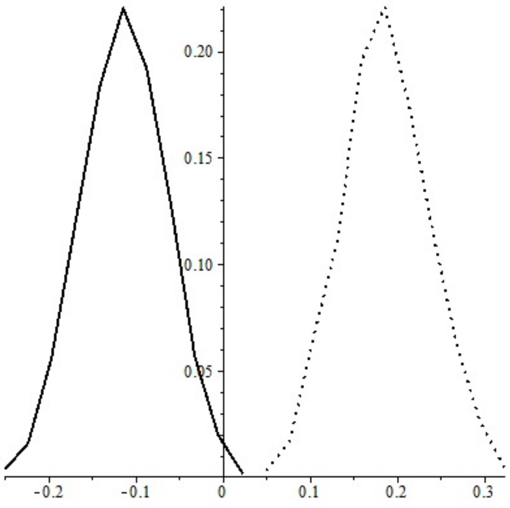
$$p^* = \frac{r-a_2}{a_3-a_2}, q^* = 1-p^*, \bar{p} = \left(1+\frac{a_3}{N}\right)p^*, \bar{q} = \left(1+\frac{a_2}{N}\right)q^*,$$

$$j_0 = \left\lfloor \log_{i_0} \frac{\alpha^*+1}{S_0\left(1+\frac{a_2}{N}\right)^N}\right\rfloor, i_0 = \frac{1+a_3/N}{1+a_2/N}.$$

2. $r \in (a_1, a_2) \cup (a_3, a_4)$,

$$C_* = \frac{1}{(1+r)^N}\left(\alpha^*+1-\left(1+\frac{r}{N}\right)^N\right)^+.$$

Example 1. Fair prices interval calculation.



Picture 1. Empirical distribution densities ($N = 2000$, firm "Magnit").

Confidence probability 0.95,
intervals $[-0.1842, 0.0015], [0.1151, 0.3118]$,
for CRR model $a = -0.0914, b = 0.2134$,
$r = 0.0583, S_0 = 4500, K = S_0, N = 10$.

Calculations for call option.

Model	Sellers successful hedging	Buyers successful hedging	Total experiments
Robust	1991	1611	2000
CRR	1343	657	2000

Calculations for option with aftereffect.

Model	Sellers successful hedging	Buyers successful hedging	Total experiments
Robust	1444	702	2000
CRR	1338	662	2000

Dependence of capital on time in the case of call option

Moment of time	Stock price	Buyers capital (robust model)	Capital (CRR model)	Sellers capital (robust model)
0	4500	1945.80	2028.91	2275.51
1	5316.95	2613.92	2667.30	2895.76
2	4826.33	1965.79	2047.83	2333.87
3	4354.47	1327.26	1458.52	1810.40
4	4967.81	1763.85	1864.01	2213.27
5	4717.86	1327.22	1465.26	1877.29
6	4238.99	650.78	871.02	1365.15
7	5013.87	1204.03	1346.51	1841.10
8	4748.88	717.05	911.76	1492.62
9	5773.92	1507.16	1601.06	2161.52
10	5214.49	699.12	798.49	1490.95

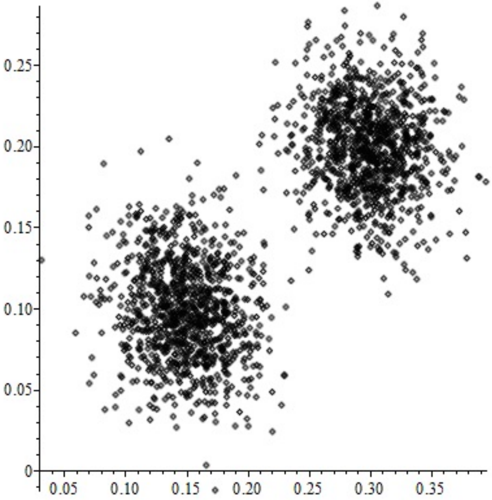
$f(S_N) = 714.49$

Dependence of capital on time in the case of option with aftereffect

Moment of time	Stock price	Buyers capital (robust model)	Capital (CRR model)	Sellers capital (robust model)
0	4500	1258.84	1287.56	1437.80
1	4958.07	1386.98	1418.63	1584.16
2	6115.55	1692.76	1732.49	1937.38
3	7269.07	1981.41	2029.90	2274.43
4	8405.76	2252.56	2310.27	2593.97
5	8291.32	2284.04	2338.28	2617.17
6	7398.59	2217.72	2257.93	2504.42
7	8786.94	2487.79	2541.84	2835.77
8	10536.58	2777.86	2852.83	3201.83
9	9282.98	2846.20	2896.21	3208.25
10	11346.56	3101.55	3179.19	3548.52

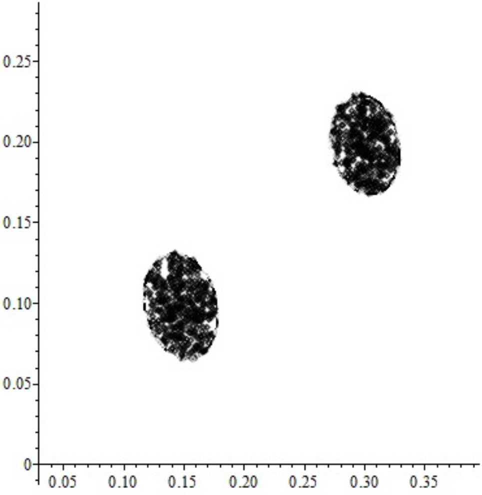
$f(S_N) = 3447.34$

Example 2. Dynamic portfolio. Two clusters.



Picture 2. Sample elements ($N = 2000$, firms "Alrosa" and "Rusolovo").

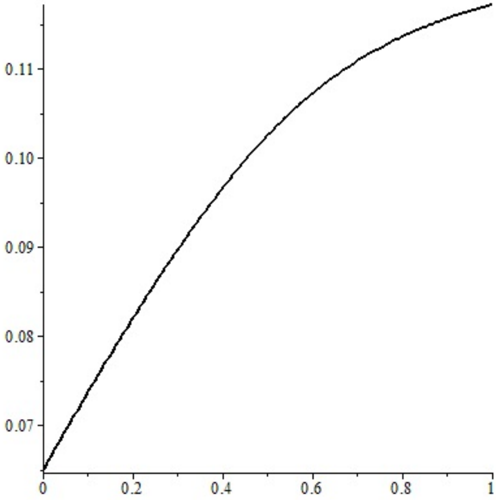
Example 2. Dynamic portfolio. Two clusters.



Picture 3. Minimal volume ellipsoids ($\eta = 0.4$).

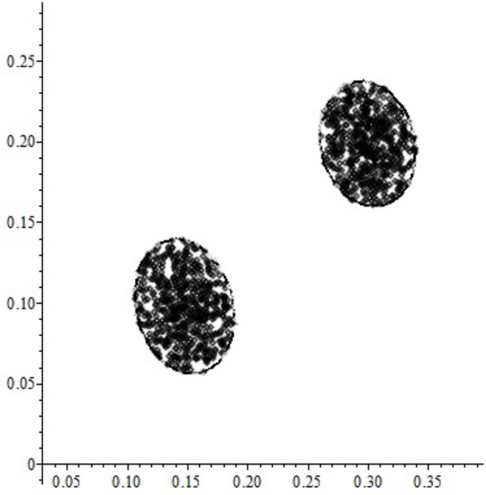
m_1, m_2 - centers, C_1, C_2 - matrices.

Example 2. Dynamic portfolio. Two clusters.



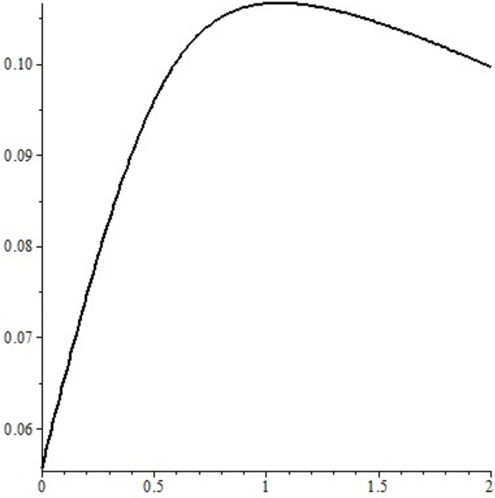
Picture 4. Return $\min((x, m_1) - \sqrt{(C_1 x, x)}, (x, m_2) - \sqrt{(C_2 x, x)})$.

Example 2. Dynamic portfolio. Two clusters.



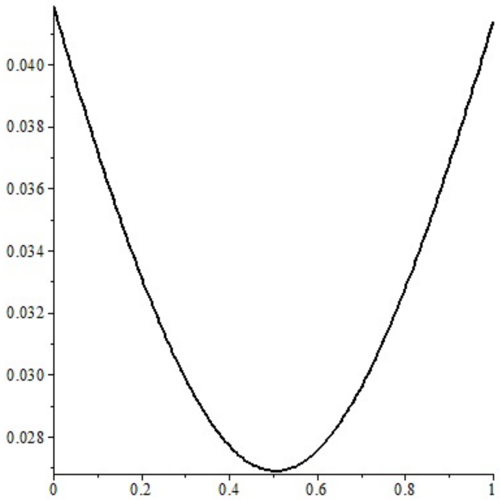
Picture 5. Minimal volume ellipsoids ($\eta = 0.6$).
m_1, m_2 - centers, C_1, C_2 - matrices.

Example 2. Dynamic portfolio. Two clusters.



Picture 6. Return $\min((x, m_1) - \sqrt{(C_1 x, x)}, (x, m_2) - \sqrt{(C_2 x, x)})$.

Example 2. Dynamic portfolio. Two clusters.

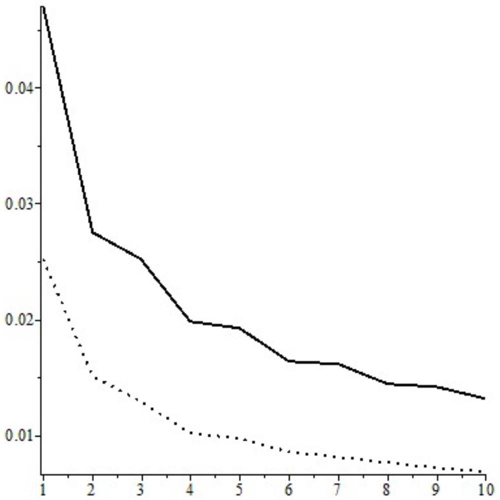


Picture 7. Risk $\max(\sqrt{(C_1x,x)}, \sqrt{(C_2x,x)})$.

Example 2. Dynamic portfolio. Two clusters.

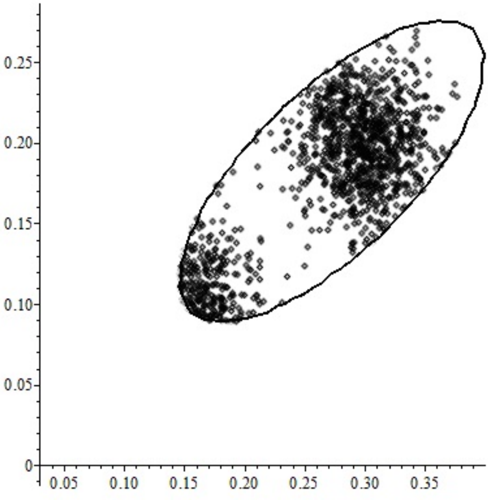
Guaranteed return $\alpha^* = \max\min((x, m_1) - \sqrt{(C_1x, x)}, (x, m_2) - \sqrt{(C_2x, x)}) = 0.11$.
Guaranteed risk $d^* = \min\max(\sqrt{(C_1x, x)}, \sqrt{(C_2x, x)}) = 0.03, x_1 = 0.51, x_2 = 0.49$.
Intervals
$a_1 = (x, m_1) - \sqrt{(C_1x, x)} = 0.10, a_2 = (x, m_1) + \sqrt{(C_1x, x)} = 0.15$,
$a_3 = (x, m_2) - \sqrt{(C_2x, x)} = 0.22, a_4 = (x, m_2) + \sqrt{(C_2x, x)} = 0.28$.
Let $\alpha^* = 0.2, r = 0.18 \in (a_2, a_3)$.

Example 2. Dynamic portfolio. Two clusters.



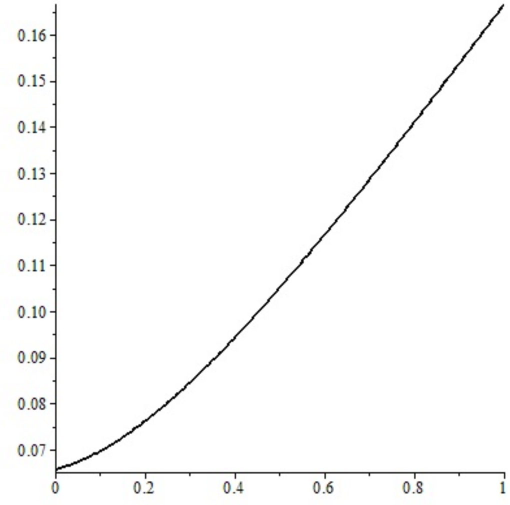
Picture 8. Dependence of sellers price and buyers price on N.

Example 2. Dynamic portfolio. Two clusters.



Picture 9. Minimal volume ellipsoid ($\eta = 0.6$).
m - center, C - matrix.

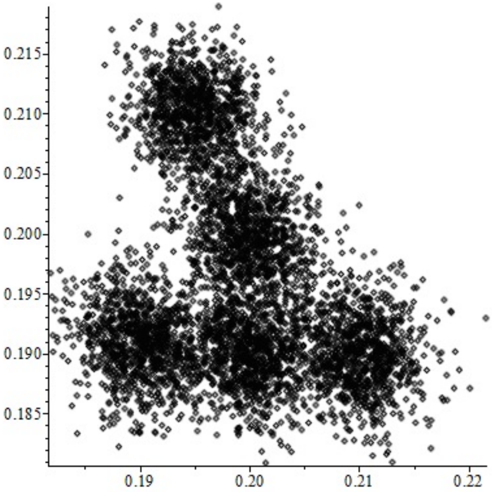
Example 2. Dynamic portfolio. Two clusters.



Picture 10. Risk $\sqrt{(Cx,x)}$.

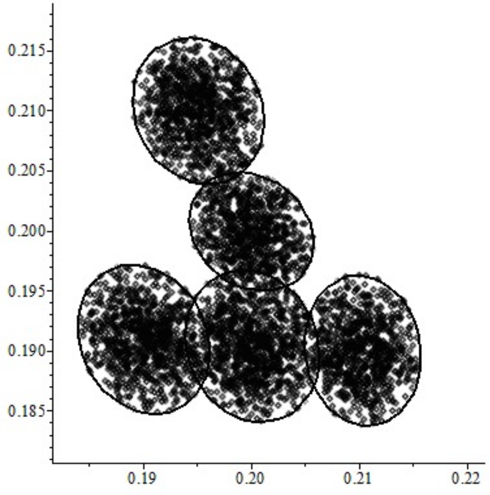
Guaranteed risk $d^* = \min\max\sqrt{(Cx,x)} = 0.07, x_1 = 0, x_2 = 1.$

Example 3. Dynamic portfolio. Five clusters.



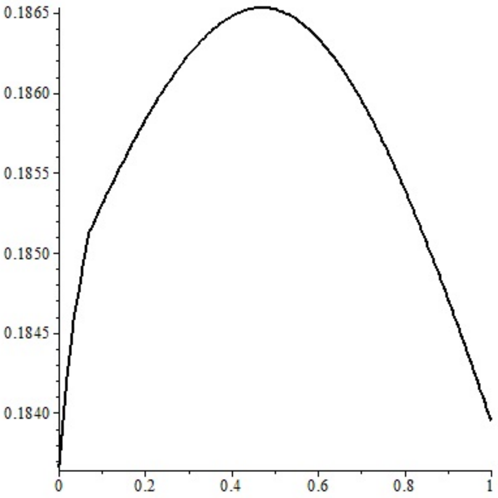
Picture 11. Sample elements ($N = 5000$, testing sample).

Example 3. Dynamic portfolio. Five clusters.



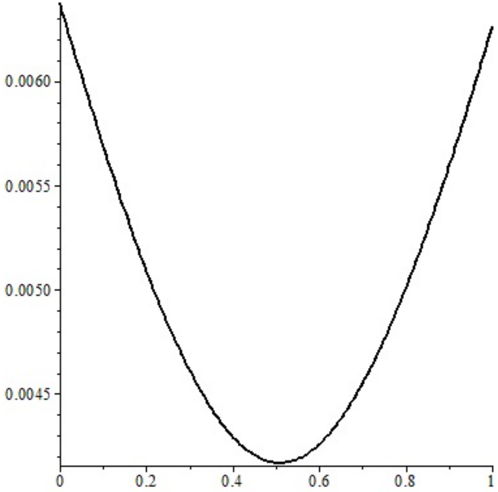
Picture 12. Minimal volume ellipsoids ($\eta = 0.9$).
$(m_i)_{i=1}^{5}$ - centers, $(C_i)_{i=1}^{5}$ - matrices.

Example 3. Dynamic portfolio. Five clusters.



Picture 13. Return $\min\limits_{i=1,\ldots,5}((x, m_i) - \sqrt{(C_i x, x)})$.

Example 3. Dynamic portfolio. Five clusters.



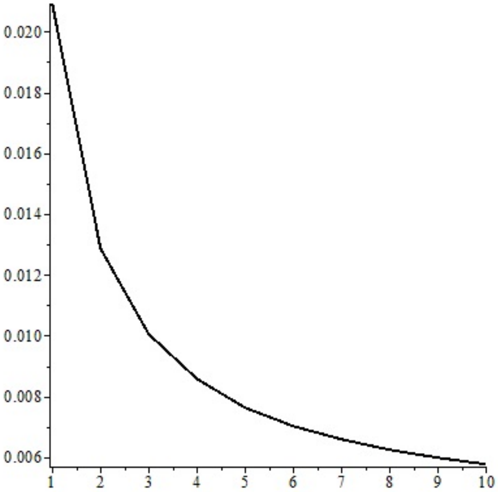
Picture 14. Risk $\max_{i=1,\ldots,5}(\sqrt{(C_i x, x)})$.

Example 3. Dynamic portfolio. Five clusters.

Guaranteed return $\alpha^* = \max \min_{i=1,\dots,5}((x, m_i) - \sqrt{(C_i x, x)}) = 0.19.$

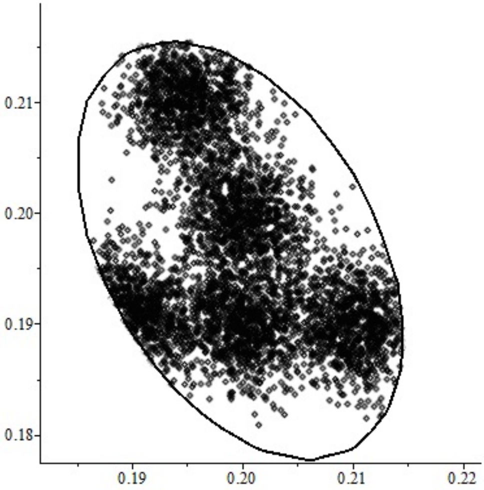
Guaranteed risk $d^* = \min \max(\sqrt{(C_i x, x)}) = 0.004, x_1 = 0.51, x_2 = 0.49.$
Interval $a_1 = 0.18$,$a_4 = 0.20.$
Let $r = 0.19.$

Example 3. Dynamic portfolio. Five clusters.



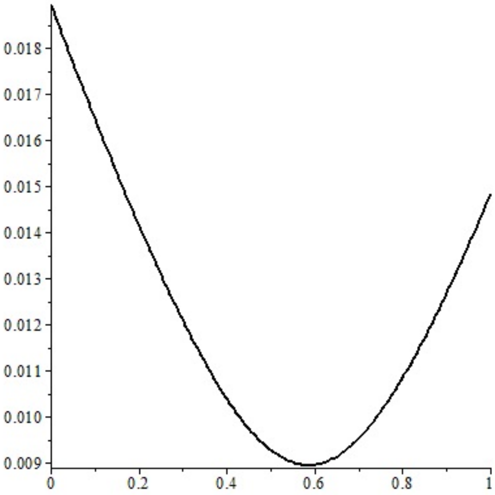
Picture 15. Dependence of fair price on N.

Example 3. Dynamic portfolio. Five clusters.



Picture 16. Minimal volume ellipsoid ($\eta = 0.6$).
m - center, C - matrix.

Example 3. Dynamic portfolio. Five clusters.



Picture 17. Risk $\sqrt{(Cx,x)}$.

Guaranteed risk $d^* = \min\max\sqrt{(Cx,x)} = 0.009, x_1 = 0.59, x_2 = 0.41.$

Example 4. Optimal portfolio. Two types of assets.

We consider sample from example 3.
$V = (\rho_i)_{i=1}^N, N = 5000, V = V_1 \cup V_2, |V_1| = |V_2|$,
V_1 - training sample, V_2 - testing sample,
$\bar{V} = \dfrac{1}{N} \sum\limits_{\rho \in V_2} (x, \rho)$ - sample average profitability,
$V_{\min} = \min\limits_{\rho \in V_2} (x, \rho)$ - sample minimal profitability,
$D = \dfrac{1}{N} \sum\limits_{\rho \in V_2} (x, \rho)^2 - (\bar{V})^2$ - sample variance.

Example 4. Optimal portfolio. Two types of assets.

Number of clusters	Optimal portfolio	$\bar{V}$	D	$V_{\min}$	λ
$K = 1$	1,0	0.998985	0.000054	0.981859	0
	0.74,0.26	0.998264	0.000028	0.983947	0.5
	0.64,0.36	0.997983	0.000023	0.984044	1
$K = 2$	0.86,0.14	0.998595	0.000037	0.983208	0
	0.86,0.14	0.998595	0.000037	0.983208	0.5
	0.86,0.14	0.998595	0.000037	0.983208	1
$K = 3$	1,0	0.999955	0.000075	0.981859	0
	1,0	0.999955	0.000075	0.981859	0.5
	0.55,0.45	0.995666	0.000022	0.983856	1
$K = 4$	0.06,0.94	0.990917	0.000031	0.982096	0
	0.56,0.44	0.993338	0.000012	0.983865	0.5
	0.56,0.44	0.993338	0.000012	0.983865	1
$K = 5$	**0,1**	**1.005529**	**0.000008**	**0.995623**	**0**
	0.46,0.54	**1.001753**	**0.000006**	**0.996051**	**0.5**
	0.47,0.53	**1.001736**	**0.000006**	**0.996083**	**1**