QUASI-CLASSICAL ASYMPTOTICS FOR THE DIRAC EQUATION

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In this paper, we consider a massless two-dimensional Dirac equation system describing the evolution of wave functions in graphene, which has the form:

$$i\varepsilon \frac{\partial u}{\partial t} = \varepsilon\left(-i\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2}\right)v + Fu,$$
$$i\varepsilon \frac{\partial v}{\partial t} = \varepsilon\left(-i\frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2}\right)u + Fv,$$

where $x \in R^2$, F is the potential, ε is a small parameter that tends to zero, and characterizes the ratio of the scales of localized inhomogeneity and the general change in the external field. Studying the asymptotics of the solution, we obtain the transmitted and reflected waves from the surface $\Phi(x,t) = 0$, where different modes can pass into each other, and also reverse waves appear when the phase and group velocities are directed in different directions.

Two tasks are considered:

1. potential $F = F\left(\frac{\Phi(x,t)}{\varepsilon}, x, t\right)$ depends on the fast variable $y = \frac{\Phi(x,t)}{\varepsilon}$ and is a smooth function, with $F(x,y,t) \to F^{\pm}(x,t)$ at $y \to \pm\infty$ is faster than any degree of y with all its derivatives. The functions of $F^{\pm}$ are also smooth. This condition reflects the localized nature of the heterogeneity. The parameter $\varepsilon \to 0$, $\Phi(x,t) : R^3 \to R$ is a smooth function, and the equation $\Phi(x,t) = 0$ defines a smooth regular hypersurface $M \subset R^3$ (heterogeneity is localized near it).

2. the potential $F(y,x,t)$ has a gap of the 1st kind on the surface of $M : \Phi(x,t) = 0$. This means that the limits of $F^{\pm}(x,t)$ are finite and are smooth functions of their arguments. In this case, the solution must be continuously on the surface of the potential gap.

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The initial conditions for both problems have the form:

$$u\Big|_{t=0} = u^0 e^{i\frac{S^0}{\varepsilon}}, \quad v\Big|_{t=0} = v^0 e^{i\frac{S^0}{\varepsilon}} \tag{1}$$

where $u^0 = u^0(x)$, $v^0 = u^0(x)$ and $S^0(x)$ are smooth functions, and u^0 and v^0 are finite, $\nabla S^0|_{\operatorname{supp} u^0} \neq 0$, $\nabla S^0|_{\operatorname{supp} v^0} \neq 0$ and $\operatorname{supp} u^0 \cap$ $\cap\, M = \varnothing$, $\operatorname{supp} v^0 \cap M = \varnothing$. The initial wave packet is located outside the localized inhomogeneity, the problem is to describe the scattering of such an initial packet by M.

References

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