Some Practical Problems of Quantum Gate Teleportation

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The Plan

TP = teleportation

1. Motivation
2. State TP
3. Gate TP (1 qubit)
4. CNOT TP
5. Toffoli TP
6. Combining gates
7. …

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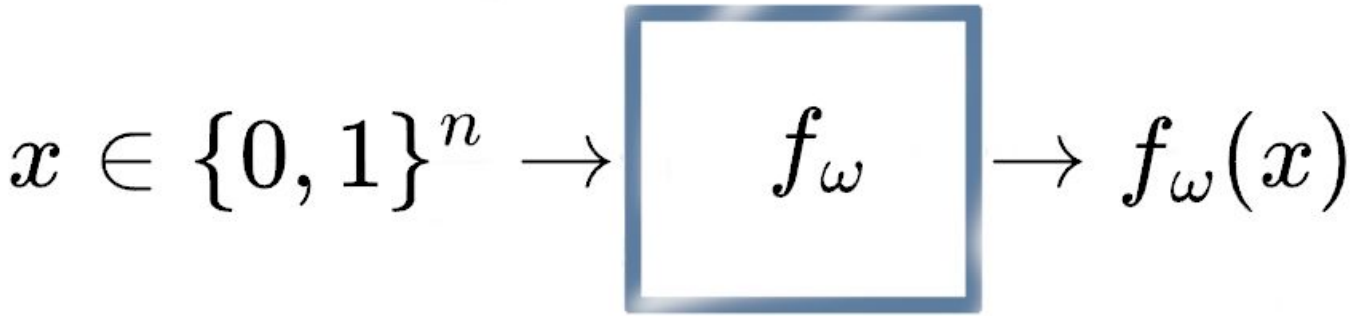
Parallel computing

$$f_\omega : \{0,1\}^n \to \{0,1\}$$

$$f_\omega(\omega) = 1$$

$$f_\omega(x \neq \omega) = 0$$



Parallel computing

$$f_\omega : \{0,1\}^n \to \{0,1\}$$

$$f_\omega(\omega) = 1$$

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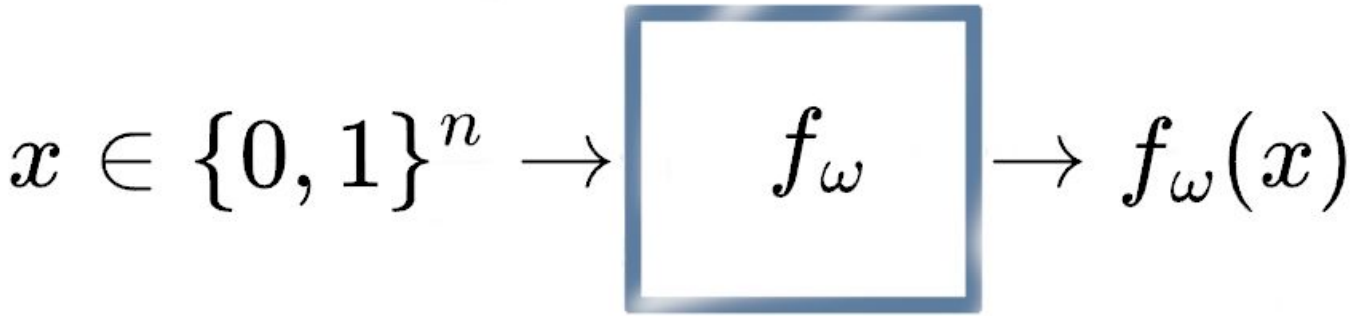
T - number of queries until success

$$E(T) = 2^{n-1}$$

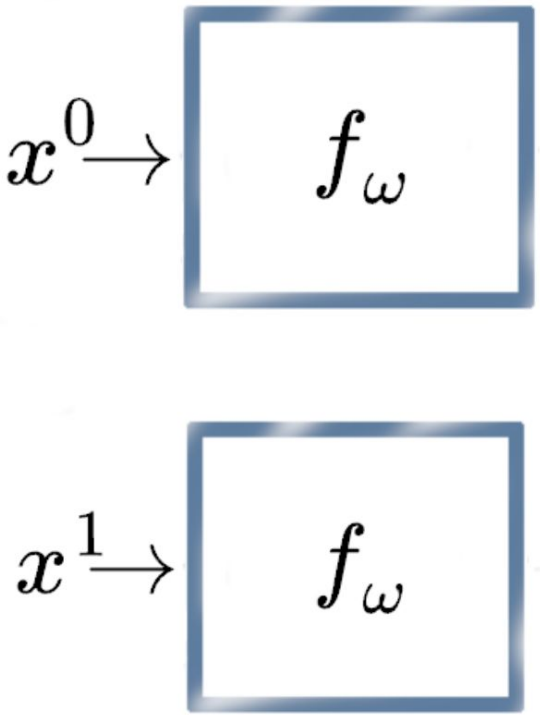


Parallel computing

$$x = x_{n-1}x_{n-1}\dots x_0$$

$$x^0 = 0x_{n-1}\dots x_0$$

$$x^1 = 1x_{n-1}\dots x_0$$



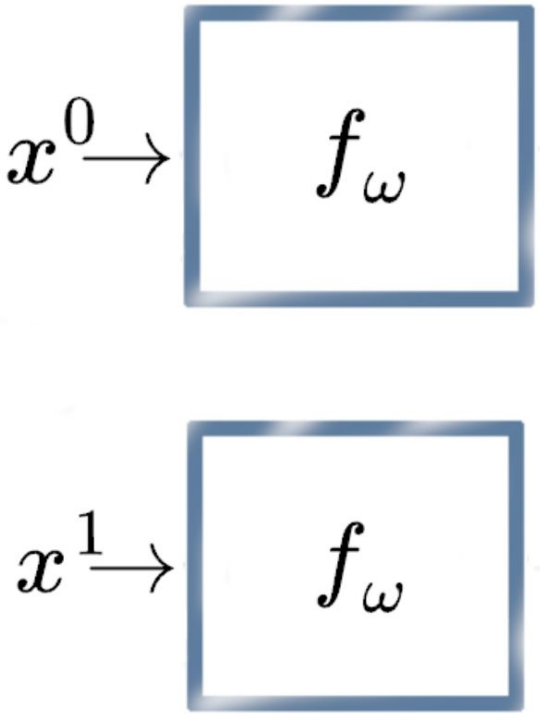
Parallel computing

$$x = x_{n-1}x_{n-1}\dots x_0$$

$$x^0 = 0x_{n-1}\dots x_0$$

$$x^1 = 1x_{n-1}\dots x_0$$

$$E(T) = 2^{n-2}$$



Parallel quantum computing

$$U_{Grov} = U_s U_\omega$$

$$U_\omega := U_{f_\omega}$$

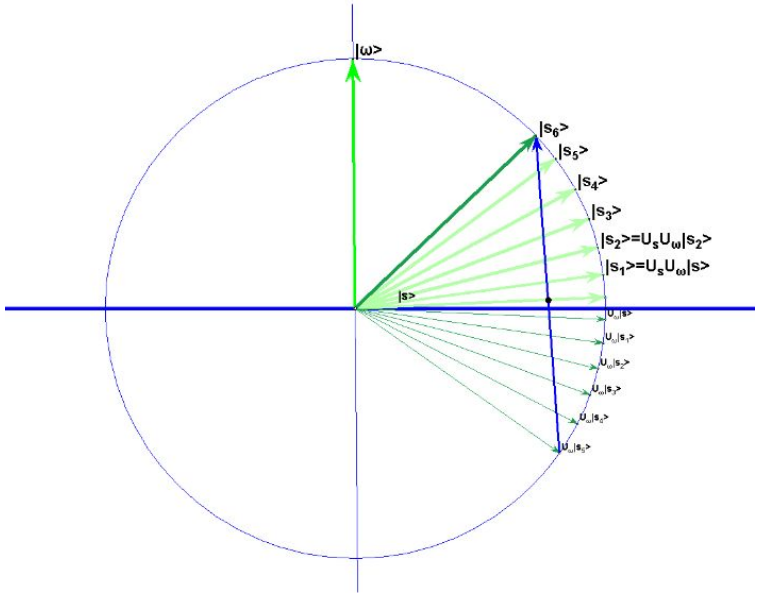
$$|x\rangle|y\rangle \to |x\rangle|y \oplus f_\omega(x)\rangle$$

$$|x\rangle|-\rangle \to (-1)^{f_\omega(x)}|x\rangle|-\rangle$$

$$U_\omega = I - 2|\omega\rangle\langle\omega|$$

$$|s\rangle = \frac{1}{2^{n/2}} \sum_{i=0}^{2^n-1} |i\rangle$$

$$U_s = 2|s\rangle\langle s| - I$$



$$N = 2^n$$

$$T \approx \frac{\pi\sqrt{N}}{4}$$

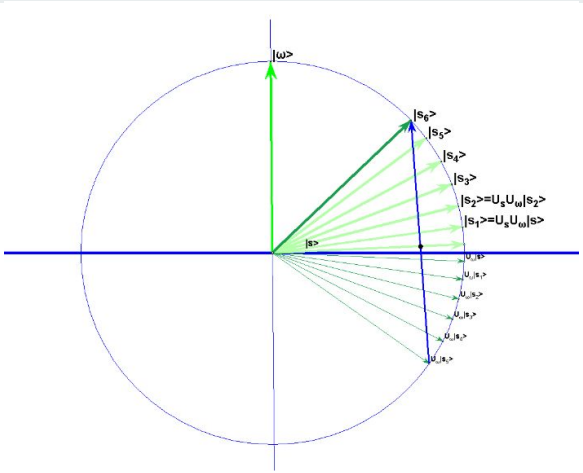
Parallel quantum computing

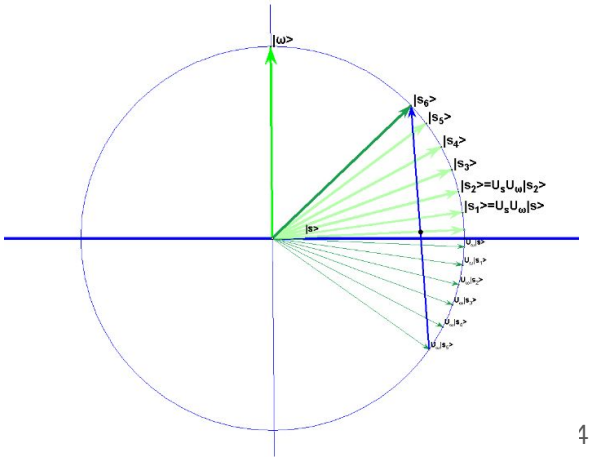
$$U_\omega^0 = U_{f_\omega^0}$$

$$U_\omega^1 = U_{f_\omega^1}$$

$$N^{0,1} = N/2$$

$$T^{0,1} = \frac{\pi\sqrt{N^{0,1}}}{4} = \frac{T}{\sqrt{2}}$$





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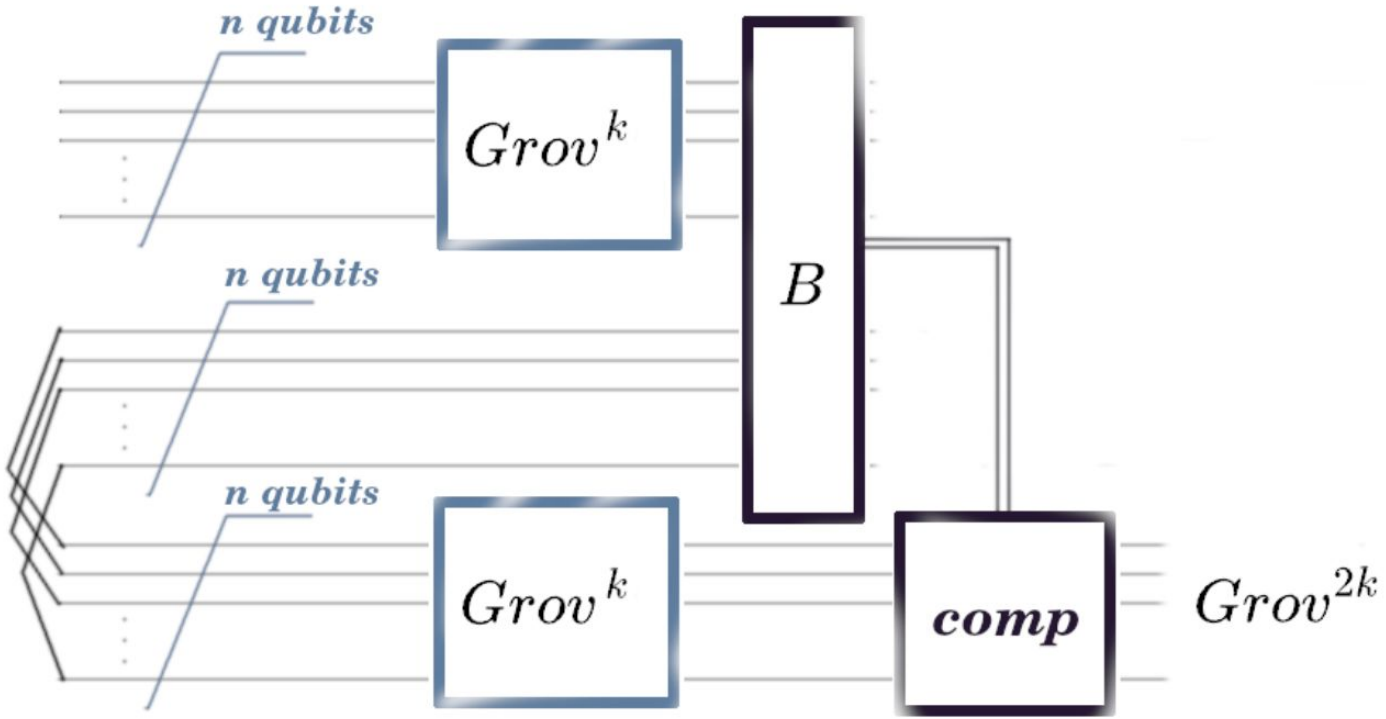

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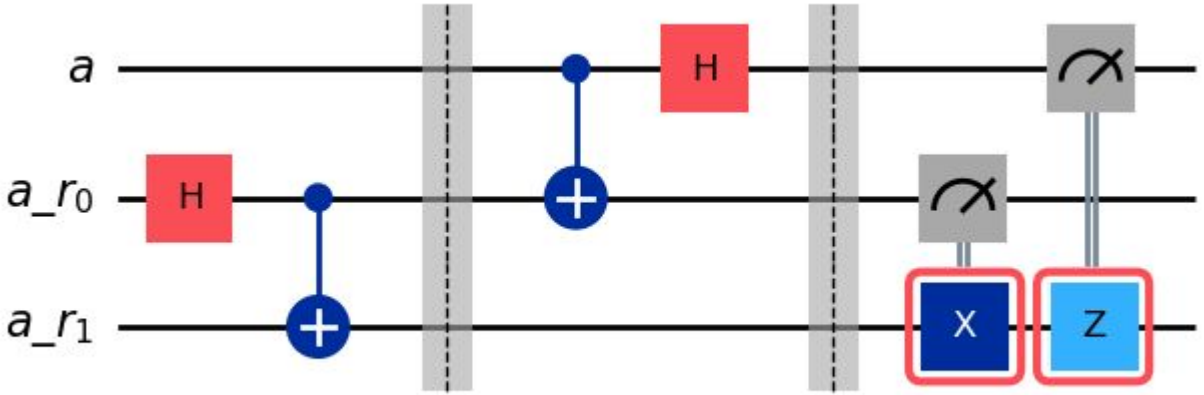
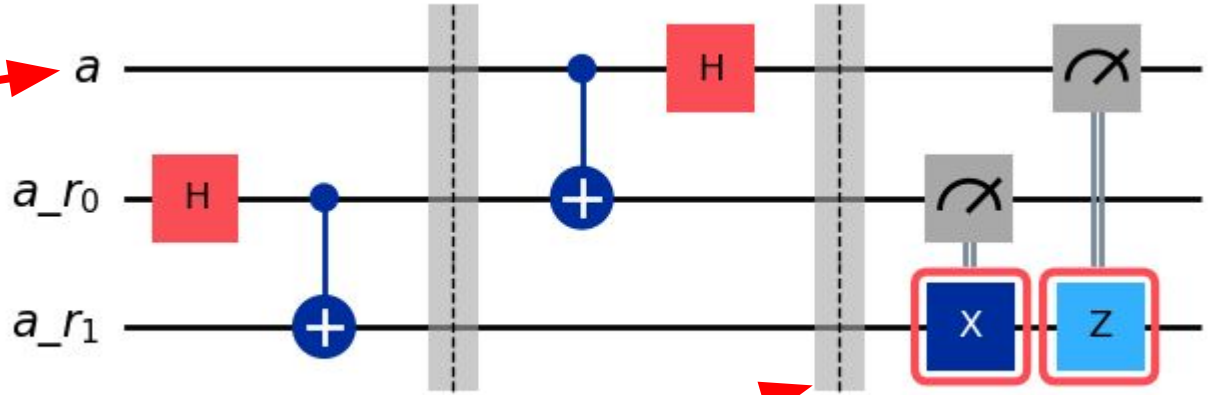
1-qubit state TP



1-qubit state TP

$$\alpha|0\rangle + \beta|1\rangle$$



$$|00\rangle(\alpha|0\rangle + \beta|1\rangle) +$$
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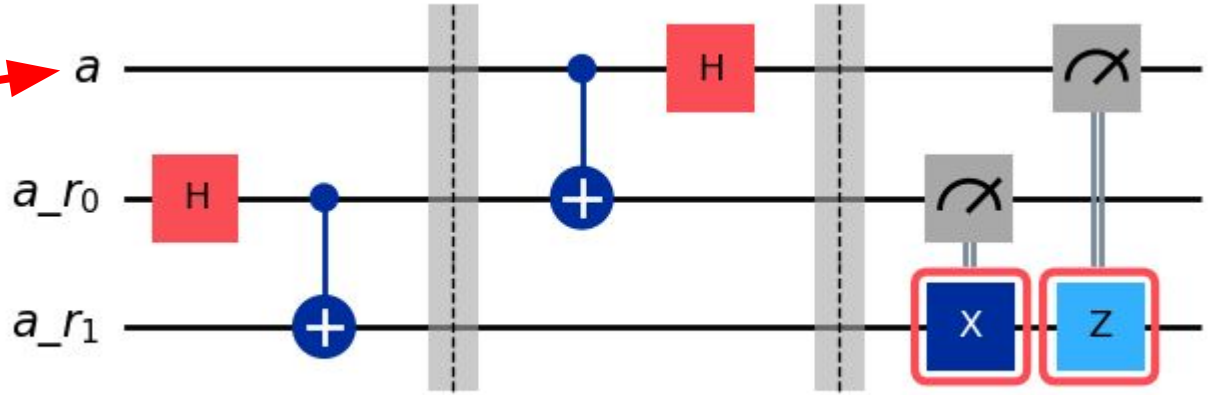

$$|00\rangle(\alpha|0\rangle + \beta|1\rangle) +$$
$$|01\rangle(\alpha|1\rangle + \beta|0\rangle) + \quad X$$
$$|10\rangle(\alpha|0\rangle - \beta|1\rangle) + \quad Z$$
$$|11\rangle(\alpha|1\rangle - \beta|0\rangle) \quad ZX$$

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1-qubit gate TP

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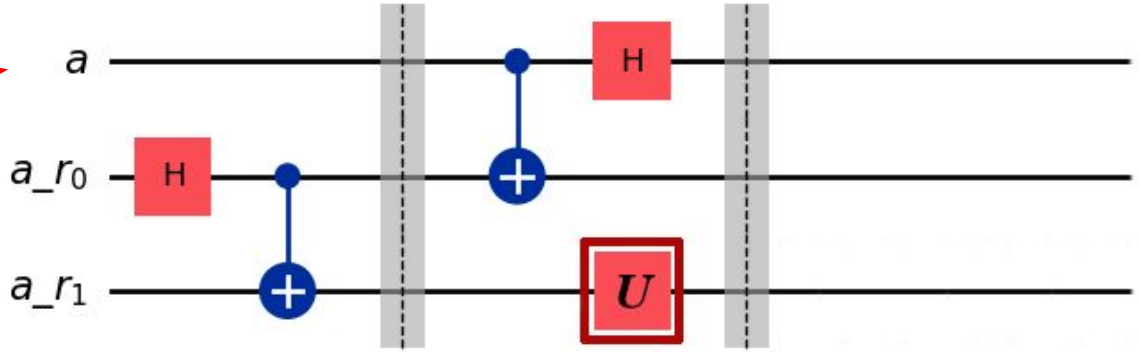

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1-qubit gate TP

$$XU|\psi\rangle \neq UX|\psi\rangle$$

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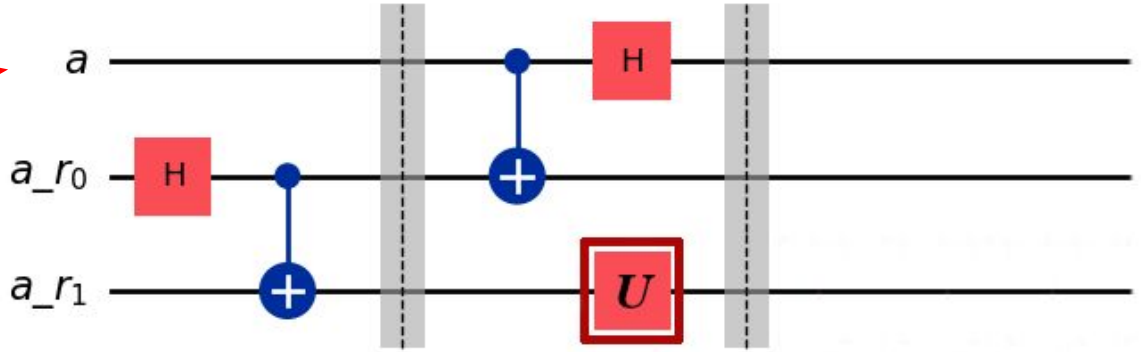

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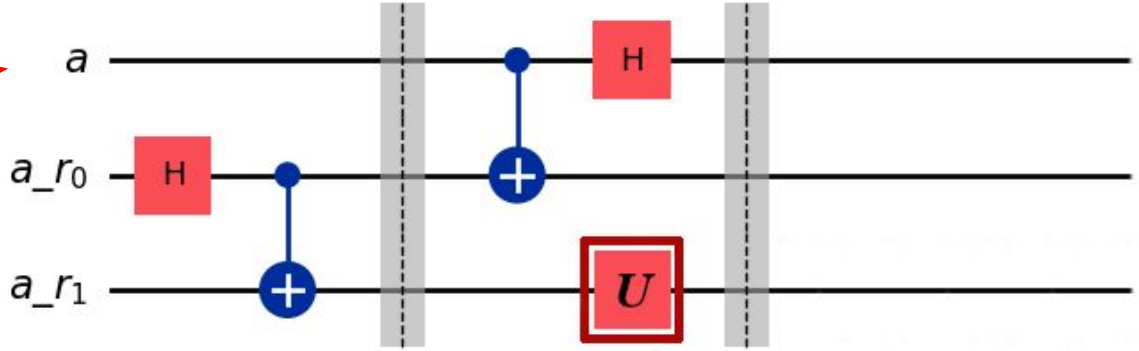

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$$UP_1U^* = P_2$$

$$P_2U = UP_1U^*U = UP_1$$

$$P_2 \rightarrow P_1$$

$$UP_1U^* = P_2$$

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$$P_2 \to P_1$$

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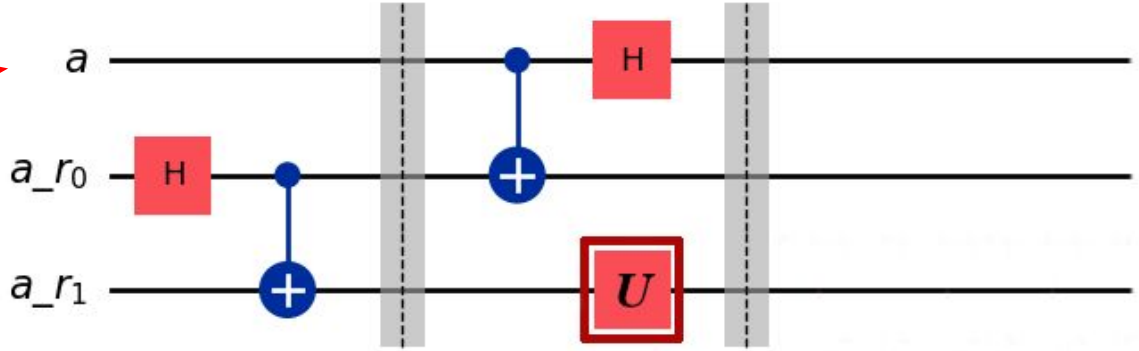

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$$ZH = HX$$

$$XH = HZ$$

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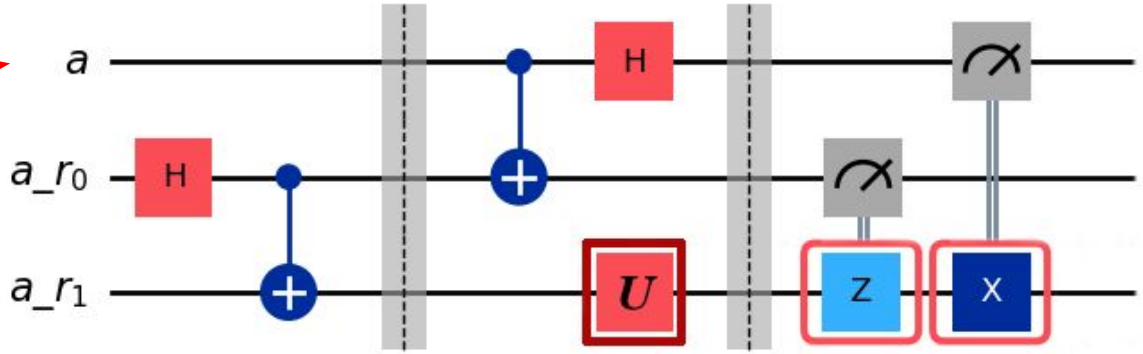

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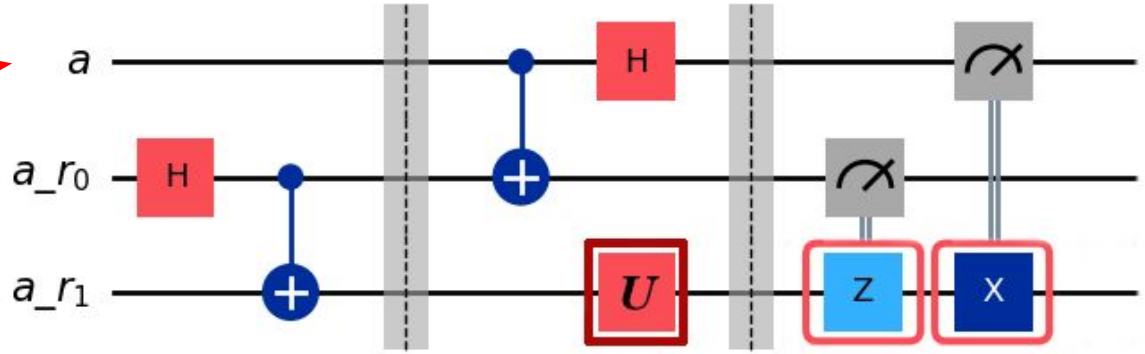

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$$UPU^* \notin \{I, X, Y, Z\}$$

$$UX \to (UXU^*)UX = U$$

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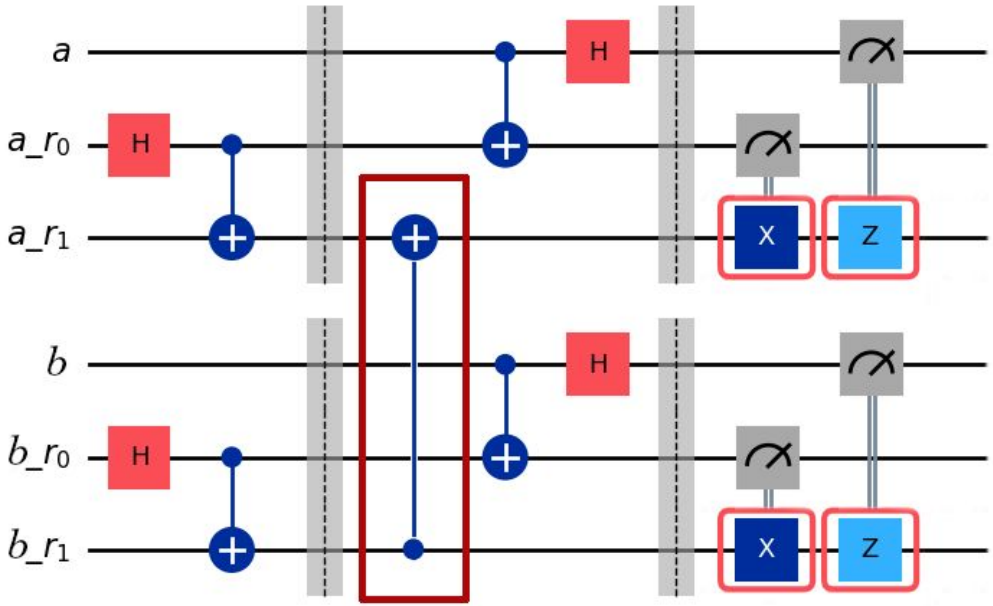
CNOT TP



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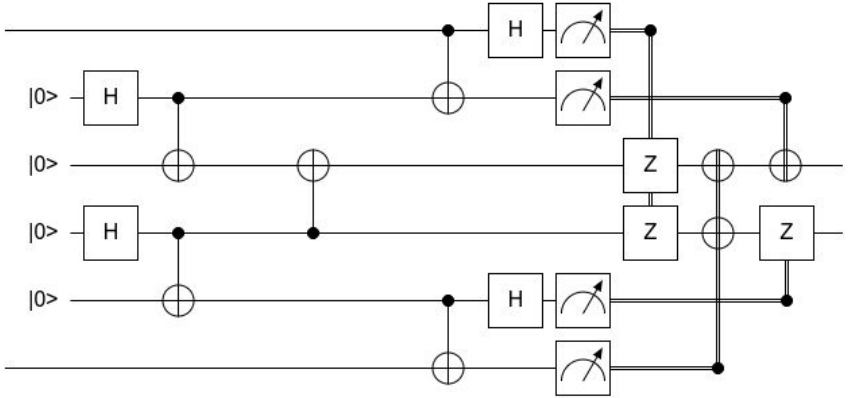

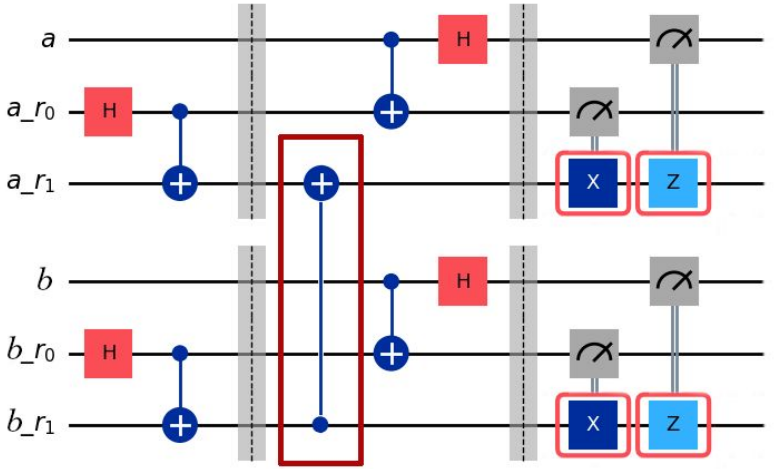

https://en.wikipedia.org/wiki/Quantum_gate_teleportation

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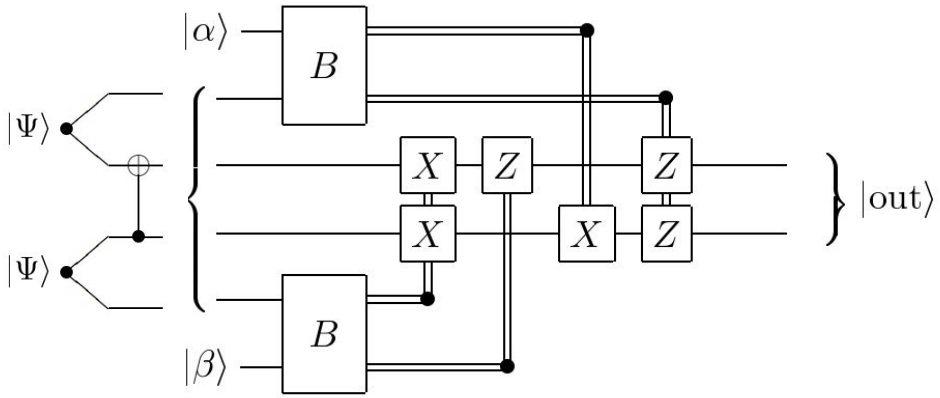
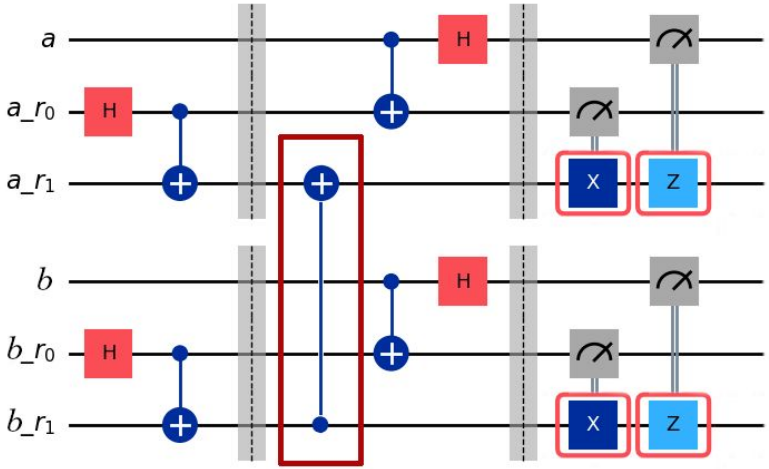
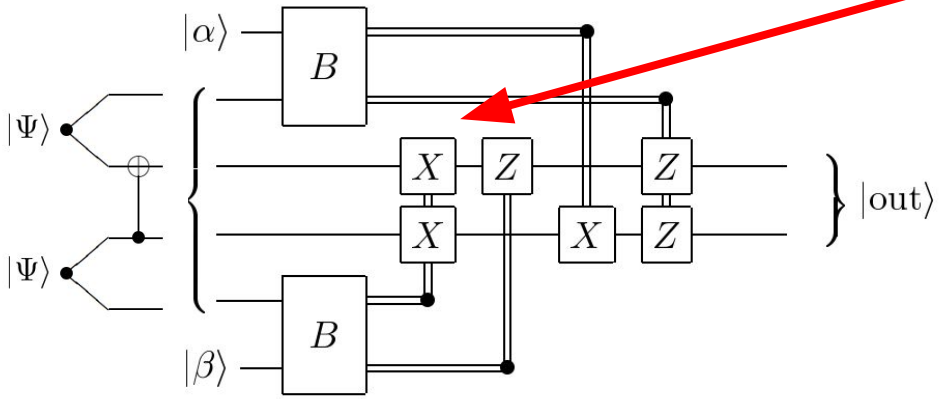


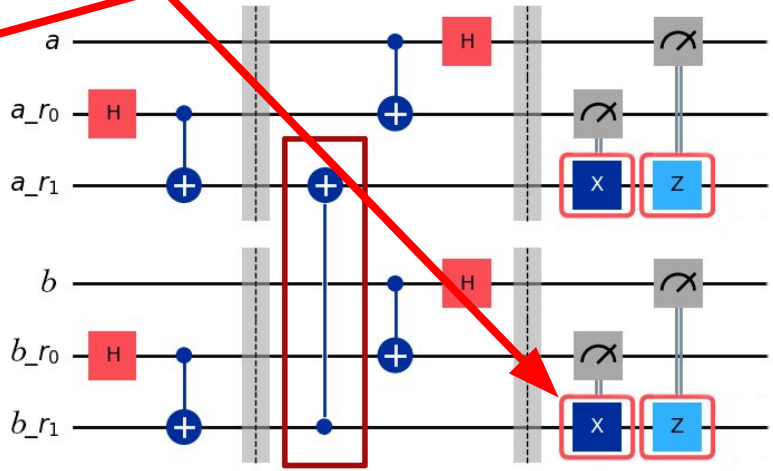


Gottesman D, Chuang IL. Quantum teleportation is a universal computational primitive. arXiv preprint quant-ph/9908010. 1999 Aug 2.

CNOT TP

$$CX(X \otimes I)CX = X \otimes X$$





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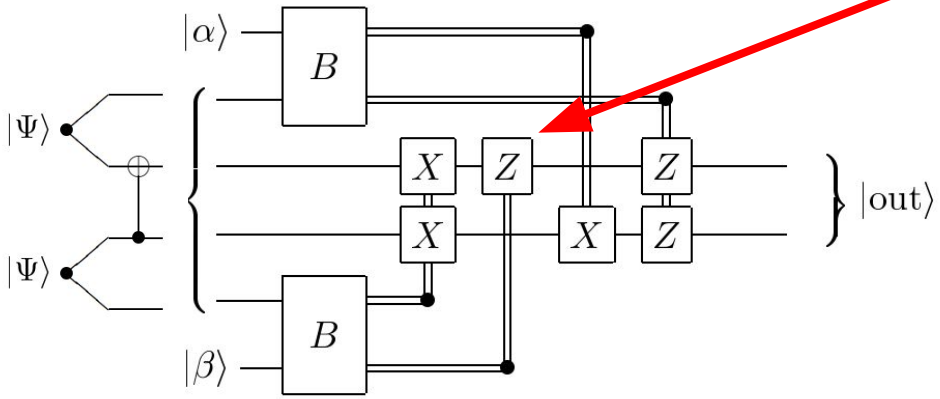
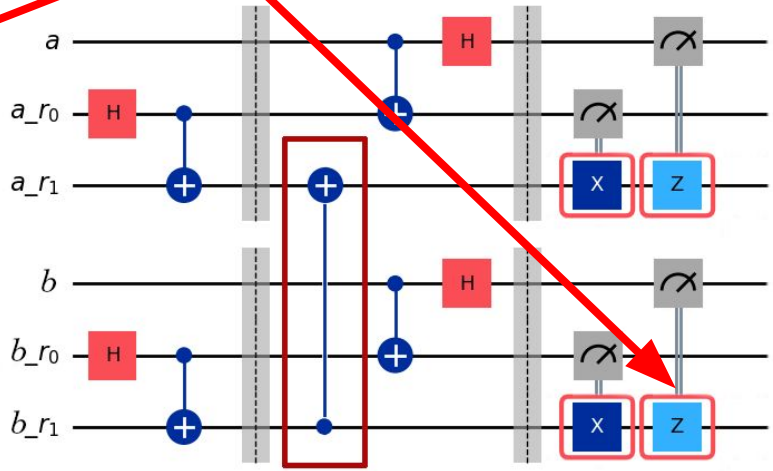
$$CX(Z \otimes I)CX = Z \otimes I$$





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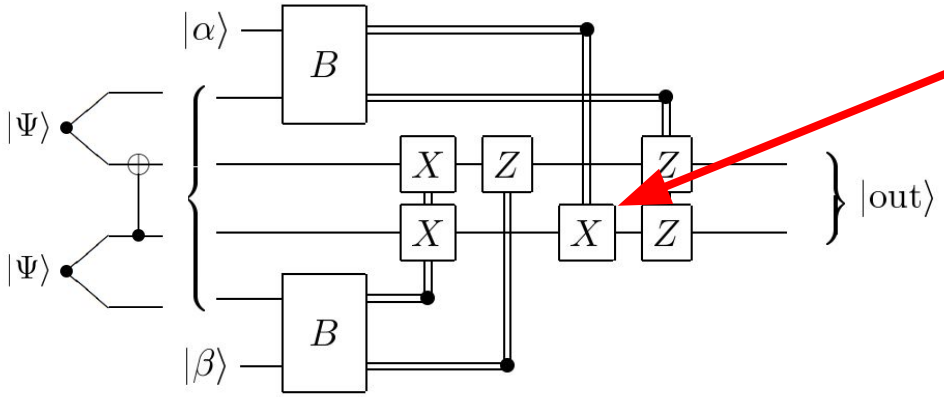

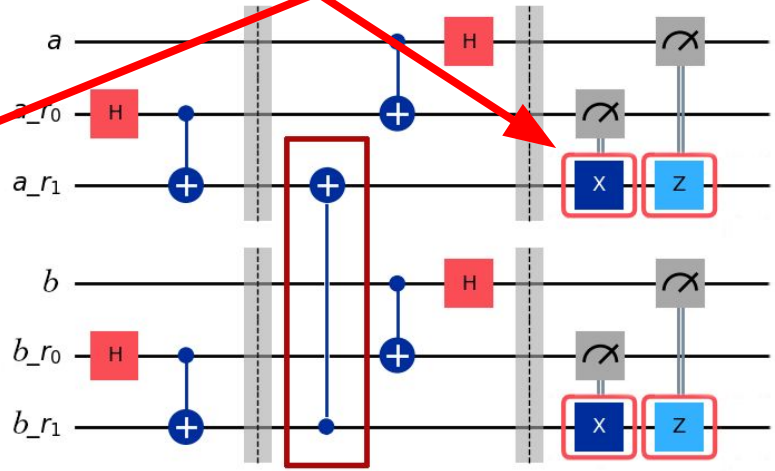

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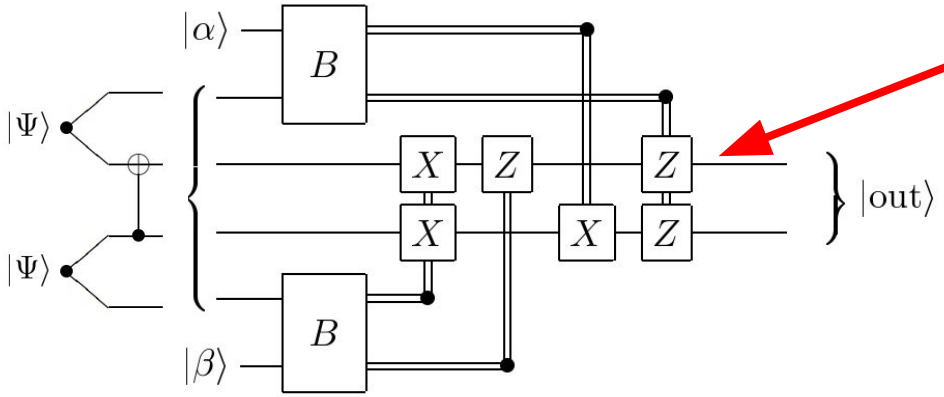
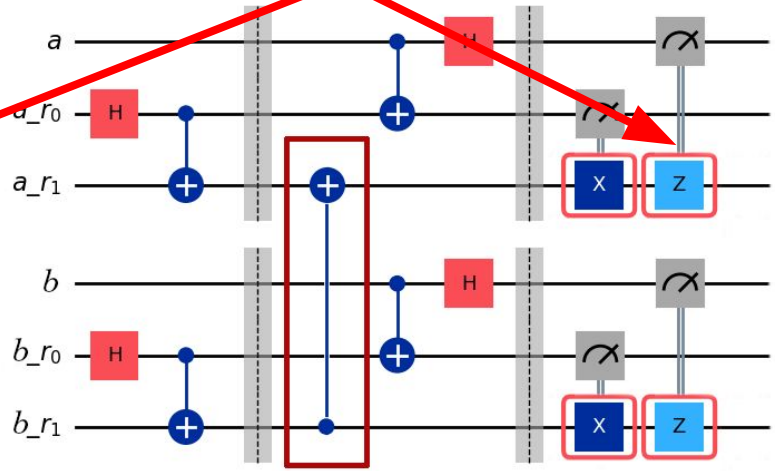
$$CX(I \otimes Z)CX = Z \otimes Z$$





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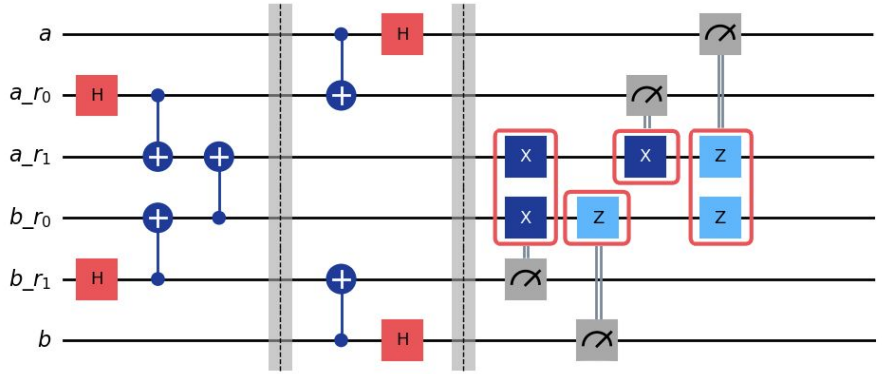

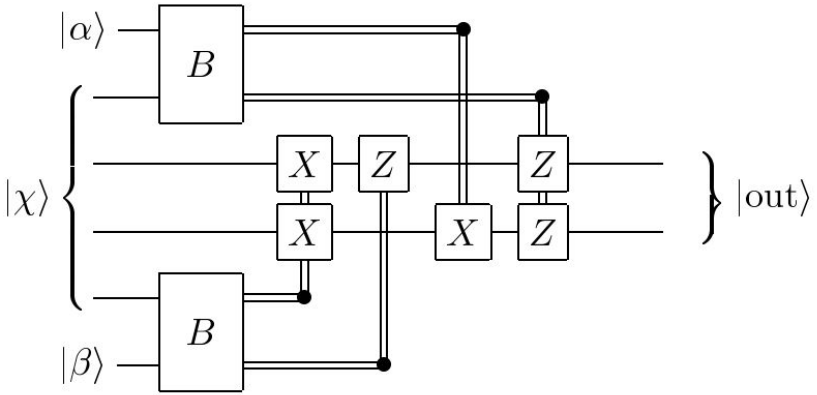

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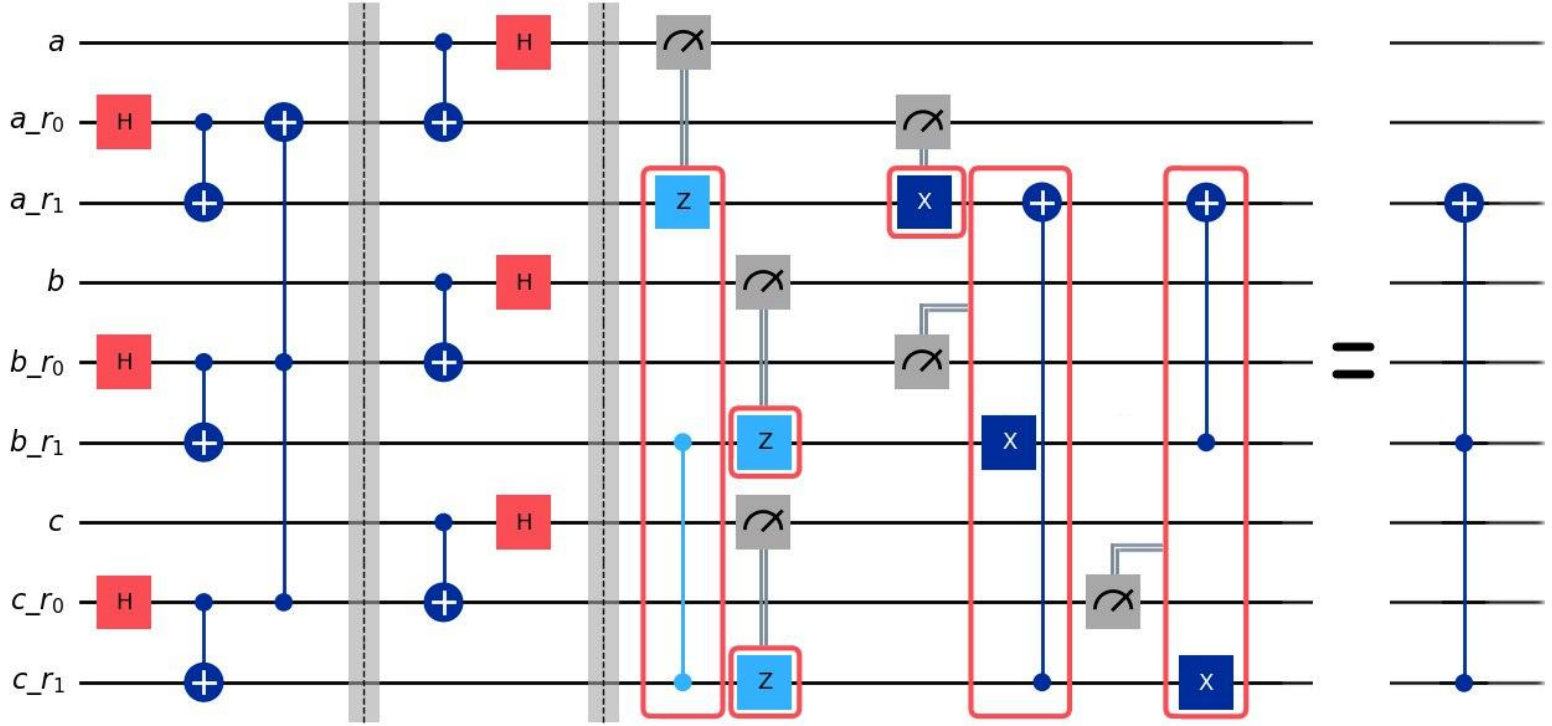
Toffoli TP



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