On the blow-up criterion for solutions of second-order quasilinear elliptic inequalities

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We consider the inequality

$$-\operatorname{div} A(x, u, \nabla u) \geq f(u) \quad \text{in } \mathbb{R}^n, \tag{1}$$

where $n \geq 2$ and A is a Carathéodory function such that

$$(A(x, s, \zeta) - A(x, s, \xi))(\zeta - \xi) \geq 0,$$

$$C_1|\xi|^p \leq \xi A(x, s, \xi), \quad |A(x, s, \xi)| \leq C_2|\xi|^{p-1}, \quad n > p > 1,$$

with some constants $C_1, C_2 > 0$ for almost all $x \in \mathbb{R}^n$ and for all $s \in \mathbb{R}$ and $\zeta, \xi \in \mathbb{R}^n$. The function f on the right-hand side of (1) is assumed to be non-negative and non-decreasing on the interval $[0, \varepsilon]$ for some $\varepsilon \in (0, \infty)$.

By a solition of (1) we mean a function $u \in W^1_{p,loc}(\mathbb{R}^n)$ such that $f(u) \in L_{1,loc}(\mathbb{R}^n)$ and

$$\int_{\mathbb{R}^n} A(x, u, \nabla u)\nabla\varphi \, dx \geq \int_{\mathbb{R}^n} f(u)\varphi \, dx$$

for any non-negative function $\varphi \in C_0^\infty(\mathbb{R}^n)$.

Theorem 1. *Inequality (1) has a positive solution such that*

$$\operatorname*{ess\,inf}_{\mathbb{R}^n} u = 0$$

if and only if

$$\int_0^\varepsilon \frac{f(t)\, dt}{t^{1+n(p-1)/(n-p)}} < \infty.$$

The proof is given in [KS24, KSS25].

[KS24] A.A. Kon'kov and A.E. Shishkov, *On global solutions of second-order quasilinear elliptic inequalities*, Math. Notes **116** (2024), pp. 1014–1019.

[KSS25] A.A. Kon'kov, A.E. Shishkov, and M.D. Surnachev, *On the existence of global solutions of second-order quasilinear elliptic inequalities*, Math. Notes (to appear in 2025).