Existence of non-radial extremal functions for Hardy–Sobolev inequalities in non-convex cones

23.06
12:00-12:30

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The symmetry breaking is obtained for Neumann problems for equations with p-Laplacian generated by the Hardy–Sobolev inequality:

$$\begin{cases} -\Delta_p u = \dfrac{u^{q-1}}{|x|^{(1-\sigma)q}} & \text{in } \Sigma_D, \\ \dfrac{\partial u}{\partial \nu} = 0 & \text{on } \partial\Sigma_D, \\ u > 0 & \text{in } \Sigma_D. \end{cases}$$

Here $1 < p < n$, $n \geqslant 3$, $0 < \sigma \leqslant 1$, $q = p^*_\sigma = \frac{np}{n-\sigma p}$, $D \subset \mathbb{S}^{n-1}$ and

$$\Sigma_D = \{xt : x \in D, t \in (0, +\infty)\} \subset \mathbb{R}^n$$

is a non-convex cone. Such problems have obvious radial solutions — Talenti–Bliss functions of $|x|$. However, under a certain restriction on the first Neumann eigenvalue $\lambda_1(D)$ of the Beltrami–Laplace operator on D:

$$\lambda_1(D) < (1-\alpha)(n-1-\alpha(p-1)), \quad \alpha = (1-\sigma)\frac{q}{p},$$

we prove this radial solution cannot be the extremal function, therefore minimizer must be non-radial.

In the case $p = 2$, $\sigma = 1$ this problem was investigated in [CPP24].

This research was supported by the Theoretical Physics and Mathematics Advancement Foundation “BASIS”.

[CPP24] G. Ciraolo, F. Pacella, and C.C. Polvara, *Symmetry breaking and instability for semilinear elliptic equations in spherical sectors and cones*, Journal de Mathématiques Pures et Appliquées **187** (2024), pp. 138–170.