The third type (Robin) boundary condition for a quasilinear problem with critical growth of the right-hand side

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Danil Bystrov

St. Petersburg State University

danil.bystrovv@gmail.com

We consider the existence of a positive solution to the following problem

$$\begin{cases} -\Delta_p u = u^{p^*-1} & \text{in } \Omega, \\ |Du|^{p-2}\partial_{\vec{n}} u + \lambda u^{p-1} = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ with $\partial\Omega \in \mathcal{C}^2$, $n \geq 2$, $1 < p < n$, $\lambda > 0$, $\Delta_p u = \operatorname{div}(|Du|^{p-2}Du)$ is the p-Laplacian operator and $p^* = \frac{np}{n-p}$ stands for the critical Sobolev exponent.

In the case of $p = 2$, some results on the solvability of (1) were obtained in [Wan91].

Since the embedding $W_p^1(\Omega) \hookrightarrow L_{p^*}(\Omega)$ is not compact, the standard variational method cannot be applied directly. We use a variant of the concentration-compactness method by P.-L. Lions and give some sharp sufficient conditions for the solvability of the problem (1).

The talk is based on joint work with A. I. Nazarov [BN24].

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[BN24] D.V. Bystrov and A.I. Nazarov, *The Robin problem for quasilinear equations with critical growth of the right-hand side*, Zapiski Nauchnykh Seminarov POMI **536** (2024), pp. 126–139.

[Wan91] X.-J. Wang, *Neumann problems of semilinear elliptic equations involving critical Sobolev exponents*, Journal of Differential Equations **93**:2 (1991), pp. 283–310.