On the existence and uniqueness of the Burgers' equation based on Ellis rheological model

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In this work, we investigate the existence and uniqueness of solutions for the viscous Burgers' equation for the isothermal flow of Ellis fluids

$$\partial_t u + u\partial_x u = \nu\partial_x \left(\phi^{-1}\left(\partial_x u\right)\right)$$

with the initial condition $u(0,x) = u_0$, $0 < x < l$, and the boundary condition $u(t,0) = u(t,l) = 0$, $0 < t < T$, where $\phi(\tau) = \left[1 + c\left|\tau\right|^{\alpha-1}\right]\tau$, $0 < c < \infty$, $0 < \alpha < \infty$, ν is the kinematic viscosity, τ is the shear stress, u is the velocity of the fluid and $l, T > 0$. We proved the existence and uniqueness of solution $u \in L^2\left(0,T;{W_0}^{1,p}(0,l)\right)$ for $u_0 \in W_0^{1,p}(0,l)$, where $p = 1 + \frac{1}{\alpha}$.

Non-Newtonian fluids, such as polymer solutions, colloidal suspensions, and biological substances, have viscosity that varies with the applied shear rate, requiring advanced models to accurately describe their flow characteristics [BAH87]. Incorporating non-Newtonian properties into Burgers' equation enables a more precise representation of fluid behavior which is crucial for applications in industrial processes, biomedical engineering and materials science. Recent studies have explored some mathematical properties of Burgers' equation with non-Newtonian viscosity, providing insight into the existence and stability of solutions for such models [Shu15]. Ellis rheological model effectively captures shear-thinning effects, where viscosity decreases as shear rate increases, a phenomenon commonly observed in polymeric and biological fluids.

[BAH87] R.B. Bird, R.C. Armstrong, and O. Hassager, *Dynamics of polymeric liquids. Vol. 1: Fluid mechanics*, John Wiley and Sons Inc., New York, NY, 1987.

[Shu15] Y. Shu, *Numerical solutions of generalized Burgers' equations for some incompressible non-Newtonian fluids*, University of New Orleans: Theses and Dissertations, 2015.