# On different statements of boundary value problems for linear and nonlinear elliptic differential–difference equations

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Olesya V. Solonukha

*FRC CSC RAS*

solonukha@yandex.ru

The relevance of study of elliptic differential-difference equations is closely connected with their important and numerous applications. Firstly, they are related to variational problems arising in the theory of multilayer plates and shells, in the theory of control of systems with aftereffect, etc. Secondly, in some cases elliptic differential–difference equations are caused by elliptic problems with non–local boundary conditions arising in plasma theory (Bitsadze-Samarskii problem), in the theory of diffusion processes, etc. For example, when considering the problem of minimum for a quadratic functional containing a function and its derivatives with shifts in spatial variables, the corresponding Euler equation has the form

$$R_Q^* A R_Q u = f, \qquad x \in Q, \tag{1}$$

where $Q \subset \mathbb{R}^n$ is a bounded domain with boundary $\partial Q \in C^\infty$, $A$ is a linear strongly elliptic differential operator with constant coefficients of the second order, $R_Q$ is a linear difference operator with constant coefficients containing shifts in spatial variables. In the case of homogeneous Dirichlet conditions $u|_{\partial Q} = 0$, this equation reduces to the form

$$A(R_Q^* R_Q)u = f, \qquad x \in Q. \tag{2}$$

Assuming $w(x) = R_Q^* R_Q u(x)$, we reduce equation (2) to the form

$$Aw = f, \qquad x \in Q. \tag{3}$$

However, the new function $w(x)$ must satisfy non–local boundary conditions when the traces of the function $w$ on some pieces of the boundary are equal to linear combinations of traces of $w$ on the shifts of these pieces inside the region. If equation (1) is derived from the problem of the minimum of a functional containing a function and its derivatives with shifts in spatial variables to the degree of $p > 1, p \neq 2$, the differential operator $A$ is nonlinear and will not commute with the difference operator. Therefore, equation (1) will not be equivalent to equation

(2), i.e. it cannot be reduced to a non-local elliptic boundary value problem. Thus, in the nonlinear case, we obtain two different types of differential-difference equations (1) and (2), which are investigated by different methods and have different applications.