

Minimax bifurcation formula for analyzing saddle-node bifurcations

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I will discuss bifurcations in nonlinear equations, focusing on a novel approach based on the minimax variational formula for identifying saddle-node bifurcations. This formula offers three significant advantages:

- (i) it allows direct detection of saddle-node bifurcations without the need to find solutions to the equations;
- (ii) it provides quantitative analysis of bifurcation values;
- (iii) offers a practical framework for calculating bifurcations.

In addition, by this formula, one can prove that the bifurcations of finite element approximations of equations converge with the corresponding bifurcations of infinite-dimensional systems.

The method will be illustrated by several examples, including:

- Identifying saddle-node bifurcation points for solutions of nonlinear elliptic systems, particularly when alternative approaches are challenging.
- Identifying bifurcation points in nonlinear finite-dimensional systems, in particular those relevant to managing complex electrical networks.
- Extension of the Perron-Frobenius theory to a set of arbitrary matrices, including those that are neither irreducible nor essentially positive, and do not preserve the cone.

The talk is based on [Ily21, Ily24, IV24, PDP20].

- [Ily21] Y. Ilyasov, *Finding saddle-node bifurcations via a nonlinear generalized Collatz–Wielandt Formula*, International Journal of Bifurcation and Chaos **31**:01 (2021), p. 2150008.
- [Ily24] Y. Il'yasov, *A finding of the maximal saddle-node bifurcation for systems of differential equations*, Journal of Differential Equations **378** (2024), pp. 610–625.
- [IV24] Y. Il'yasov and N. Valeev, *An extension of the Perron-Frobenius theory to arbitrary matrices and cones*, The Electronic Journal of Linear Algebra **40** (2024), pp. 788–802.

- [PDP20] P.D.P. Salazar, Y. Ilyasov, L.F.C. Alberto, E.C.M. Costa, and M.B.C. Salles, *Saddle-node bifurcations of power systems in the context of variational theory and nonsmooth optimization*, IEEE Access **8** (2020), pp. 110986–110993.