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A posteriori analysis of nonlinear boundary value problems with monotone operators

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Methods for obtaining fully computable estimates of the distance between a given function and a solution to a boundary value problem are discussed. They are based on special functional identities that hold for boundary value and initial boundary value problems with monotone operators. The left-hand side of the identity represents a certain measure of the distance between the approximate and exact solutions. This measure is dictated by the differential operator. It is a natural characteristic of the closeness of a function to the solution of the problem in question. In particular, minimizing sequences of variational problems converge to the minimizer with respect to precisely this measure. In some cases, the right-hand side of the identity contains only known data of the problem and functions characterizing the approximate solution. Then, the identity can be directly used for measuring errors of numerical approximations. In other cases, the right hand side contains unknown functions, which, however, can be excluded. As a result we obtain guaranteed two–sided error bounds.

Estimates allow us to estimate the error of any approximation of problems regardless of the method by which they have been constructed. In addition, they allow comparison of exact solutions to problems with different data, which makes it possible to evaluate errors in mathematical models. For example, such type errors arise in dimension reduction, homogenization, and simplification of mathematical models based on partial differential equations.