26.06
15:30-16:00

On maximum and comparison principles for parabolic problems with the p-Laplacian

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We will discuss strong and weak versions of maximum and comparison principles for solutions of a class of problems

$$\begin{cases} \partial_t u - \Delta_p u = \lambda |u|^{p-2}u + f(x,t), & (x,t) \in \Omega_T, \\ u(x,0) = u_0(x), & x \in \Omega, \\ u(x,t) = 0, & (x,t) \in \partial\Omega \times (0,T), \end{cases} \tag{$\mathcal{P}$}$$

where $\lambda \in \mathbb{R}$, $\Omega_T := \Omega \times (0,T)$ is a parabolic cylinder, $\Omega \subset \mathbb{R}^N$ $(N \geq 1)$ is a bounded domain with Lipschitz boundary $\partial\Omega$, and $T \in (0,+\infty)$.

It is well-known that in the linear case $p = 2$ any (classical) solution u of ($\mathcal{P}$) satisfies the weak maximum principle, that is, the assumptions $u_0 \geq 0$ in Ω and $f \geq 0$ in Ω_T imply that $u \geq 0$ in Ω_T. Moreover, the additional assumption $u(x_0,t_0) = 0$ for some $(x_0,t_0) \in \Omega_T$ yields $u \equiv 0$ in Ω_{t_0}, i.e., the strong maximum principle holds. At the same time, analogous principles for $p \neq 2$ cannot be satisfied, in general, without additional assumptions on the parameter λ, initial and source data; they are significantly different for the *fast diffusion* (singular case, $p < 2$) and *slow diffusion* (degenerate case, $p > 2$).

Several related counterexamples are given. The talk is based on the works [BT14, BT19].

[BT14] V. E. Bobkov and P. Takáč, *A strong maximum principle for parabolic equations with the p-Laplacian*, Journal of Mathematical Analysis and Applications **419**:1 (2014), pp. 218–230.

[BT19] V. Bobkov and P. Takáč, *On maximum and comparison principles for parabolic problems with the p-Laplacian*, Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas **113** (2019), pp. 1141–1158.