Nonlinear elliptic variational inequalities with contacting and non-contacting measurable bilateral obstacles

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We consider variational inequalities with invertible operators $\mathcal{A}_s\colon W_0^{1,p}(\Omega) \to W^{-1,p'}(\Omega)$, $s \in \mathbb{N}$, in divergence form and constraint set $V \subset W_0^{1,p}(\Omega)$ defined by a measurable lower obstacle $\varphi\colon \Omega \to \overline{\mathbb{R}}$ and a measurable upper obstacle $\psi\colon \Omega \to \overline{\mathbb{R}}$, where Ω is a nonempty bounded open set in $\mathbb{R}^n$ $(n \geqslant 2)$ and $p > 1$.

It is assumed that the sequence $\{\mathcal{A}_s\}$ G-converges to an invertible operator $\mathcal{A}\colon W_0^{1,p}(\Omega) \to W^{-1,p'}(\Omega)$. For the obstacles φ and ψ, some different cases are considered.

In the first case, it is assumed that, for every nonempty open set ω in $\mathbb{R}^n$ with $\overline{\omega} \subset \Omega$, there exist functions $\varphi_\omega, \psi_\omega \in W_0^{1,p}(\Omega)$ such that $\varphi \leqslant \varphi_\omega \leqslant \psi_\omega \leqslant \psi$ a.e. in Ω and $\varphi_\omega < \psi_\omega$ a.e. in ω. In this case, we have $\operatorname{meas}\{\varphi = \psi\} = 0$.

In the second case, it is assumed that the following conditions are satisfied:

(C_1) $\operatorname{int}\{\varphi = \psi\} \neq \varnothing$ and $\operatorname{meas}(\partial\{\varphi = \psi\} \cap \Omega) = 0$;

(C_2) there exist functions $\bar{\varphi}, \bar{\psi} \in W_0^{1,p}(\Omega)$ such that $\varphi \leqslant \bar{\varphi} \leqslant \bar{\psi} \leqslant \psi$ a.e. in Ω and $\operatorname{meas}(\{\varphi \neq \psi\} \setminus \{\bar{\varphi} \neq \bar{\psi}\}) = 0$.

In this case, we have $\operatorname{meas}\{\varphi = \psi\} > 0$.

Finally, in the third case, it is assumed that $\varphi \leqslant 0$ and $\psi \geqslant 0$ a.e. in Ω. This case admits both possibilities: $\operatorname{meas}\{\varphi = \psi\} = 0$ and $\operatorname{meas}\{\varphi = \psi\} > 0$. Therein, an additional condition on the coefficients of the operators $\mathcal{A}_s$ is required.

We expose our recent results showing that in all the described cases, the solutions u_s of the considered variational inequalities converge weak-ly in $W_0^{1,p}(\Omega)$ to the solution u of a similar variational inequality with the operator $\mathcal{A}$ and the constraint set V. We note that in the first and third cases, $\mathcal{A}_s u_s \to \mathcal{A}u$ strongly in $W^{-1,p'}(\Omega)$, while in the second case, this is not true in general. Furthermore, in the second case, the sequence of energy integrals $\langle \mathcal{A}_s u_s, u_s\rangle$ does not converge to $\langle \mathcal{A}u, u\rangle$ in general.