Very singular and large solutions of semilinear elliptic and parabolic equations

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The history of the study of two classes of singular solutions of the stationary and non-stationary diffusion-nonlinear absorption type equations is discussed. The first class are large solutions, i.e. non-negative solutions in bounded domain that take infinitely large value on the entire boundary of the corresponding domain. The existence of such solutions and the study of their properties were initiated by the works of L. Bieberbach (1916), J.B. Keller (1957), R. Osserman (1957), C. Loevner-L. Nirenberg (1974). The second class of solutions are very singular solutions, i.e. solutions that have singularities on the boundary of the corresponding domain that are stronger than those admissible for the corresponding linear equations. Such solutions were first established in the semilinear heat equations by H. Brezis-L. Peletier-D. Terman (1986). The studies of these classes of solutions were continued by many leading mathematicians.

In the recent decades, the properties of large and very singular solutions have been actively studied in the case of general absorption nonlinearities degenerating on the boundaries of corresponding domains. Exact conditions on the mentioned degeneration have been found, ensuring the existence of solutions from the described classes. Exact conditions for the uniqueness of large solutions have been obtained, which surprisingly "almost" coincide with the conditions for the existence of very singular solutions. The report will present new results on the existence, uniqueness and asymptotic properties of mentioned classes of solutions.