Optimization inverse spectral problems and nonlinear differential operators

27.06
11:30-12:00

Nur F. Valeev

Institute of Mathematics with Computing Centre, Ufa Federal Research Centre, RAS

valeevnf@gmail.com

The report addresses a new formulation of the inverse spectral problem for self-adjoint semi-bounded differential operators (both for partial and ordinary derivatives) — the optimization inverse spectral problem.

In the optimization inverse spectral problem, the spectral data of the operator $L(q_1, q_2, \ldots, q_n)$, where $q_1(x), q_2(x), \ldots, q_n(x)$ are the coefficients of the corresponding differential expression, consist of only a part of the spectrum, specifically a finite number of eigenvalues:

$$\lambda_1^* < \lambda_2^* < \cdots < \lambda_m^* \in \mathbb{R}.$$

Based on these spectral data, it is required to find new coefficients $\hat{q}_1, \hat{q}_2, \ldots, \hat{q}_n$ that are minimally distant (in some metric) from the given $q_1, q_2, \ldots, q_n$, such that:

$$\lambda_k(L(\hat{q}_1, \hat{q}_2, \ldots, \hat{q}_n)) = \lambda_k^*, \quad k = 1, \ldots, m.$$

Such problems are hereafter referred to as “optimization inverse spectral problems with incomplete data” (abbreviated as OISP).

The formulation of the optimization inverse spectral problem naturally arises in various mathematical models for the identification and construction of linear dynamical systems with specified resonance characteristics.

A notable and certainly noteworthy feature of optimization inverse spectral problems is that each such problem is equivalent to some nonlinear differential operator. Moreover, by selecting the linear operator $L(q_1, q_2, \ldots, q_n)$ and the functional spaces for the coefficients $q_1(x), q_2(x), \ldots, q_n(x)$ in the formulation of OISPs, one can “construct” functionals that generate various nonlinear equations. In particular, it is possible to obtain well-known nonlinear equations and operators in mathematical physics: nonlinear (systems of) Schrödinger equations, Gross-Pitaevskii equations, Hartree-Fock-type systems, p-Laplace operators, and others.

Using the example of Schrödinger operators, the report will present the general ideas and approaches to studying the optimization inverse spectral problem, as well as results on the existence of solutions, cases of solution uniqueness, and the representation of optimal coefficients. The main focus will be on describing the nontrivial connection between the investigated inverse problem and the nonlinear Schrödinger equation (NLSE), the Gross-Pitaevskii equation, Manakov-type systems, and the study of certain properties of these equations (existence, uniqueness, nodality, and others).

[IV19] Y.Sh. Ilyasov and N.F. Valeev, *On nonlinear boundary value problem corresponding to N-dimensional inverse spectral problem*, Journal of Differential Equations **266**:8 (2019), pp. 4533–4543.

[IV21] Y. Ilyasov and N. Valeev, *Recovery of the nearest potential field from the m observed eigenvalues*, Physica D: Nonlinear Phenomena **426** (2021), p. 132985.

[Sad24] Sadovnichii, V.A. and Sultanaev, Ya.T. and Valeev, N.F., *Optimization inverse spectral problem for the one-dimensional Schrödinger operator on the entire real line*, Differential Equations **60**:4 (2024), pp. 465–471.

[SSV22] V.A. Sadovnichii, Ya.T. Sultanaev, and N.F. Valeev, *Optimization inverse spectral problem for a vector Sturm–Liouville operator*, Differential Equations **58**:12 (2022), pp. 1694–1699.

[VI20] N.F. Valeev and Y.Sh. Ilyasov, *Inverse spectral problem for Sturm–Liouville operator with prescribed partial trace*, Ufa Math. J **12**:4 (2020), pp. 20–30.