

On the weighted Neumann eigenvalue problem in Hölder domains

Valerii Pchelintsev

Tomsk State University

va-pchelintsev@yandex.ru

We consider the weighted Neumann (p, q) -eigenvalue problem [GPU24]:

$$-\operatorname{div}(|x|^\alpha |\nabla u|^{p-2} \nabla u) = \lambda \|u\|_{L_q(\Omega_\gamma)}^{p-q} |u|^{q-2} u \text{ in } \Omega, \quad \frac{\partial u}{\partial \nu} = 0 \text{ on } \partial\Omega_\gamma,$$

in bounded Hölder γ -singularities domains $\Omega_\gamma \subset \mathbb{R}^n$.

In the case $1 < p < \alpha + \gamma$ and $1 < q < p^* = \gamma p / (\alpha + \gamma - p)$ where $0 < \alpha < n(p-1)$, $n \geq 2$, we prove solvability of this eigenvalue problem and existence of the minimizer of the associated variational problem. In addition, we establish some regularity results of the eigenfunctions and some estimates of (p, q) -eigenvalues.

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[GPU24] P. Garain, V. Pchelintsev, and A. Ukhlov, *On (p, q) -eigenvalues of the weighted p -Laplace operator in outward Hölder cuspidal domains*, *Revista Matemática Complutense* (2024), pp. 1–24.