Homogenization problems of Navier-Stokes equation for the domain perforated along the boundary

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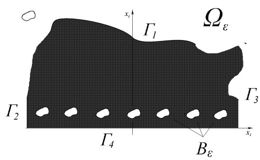
Murad Apayev

Lomonosov Moscow State University

apayevmr@gmail.com

We describe the domain. Let Ω denote a bounded domain in $\mathbb{R}^2$, lying in the upper half-plane, whose boundary Γ is piecewise smooth and consists of several parts: $\Gamma = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup \Gamma_4$, where Γ_4 is the segment $\left[-\frac{1}{2}; \frac{1}{2}\right]$ on the abscissa axis, Γ_2 and Γ_3 belong to the lines $x_1 = -\frac{1}{2}$ and $x_1 = \frac{1}{2}$, respectively, $\Gamma \backslash \Gamma_4$ is smooth. Here and throughout $\varepsilon = \frac{1}{2\mathcal{N}+1}$ is the small parameter, $\mathcal{N}$ is a natural number, $\mathcal{N} \gg 1$.

We use the following notation. Let G be an arbitrary two-dimensional domain with a smooth boundary lying in the circle $K = \{\xi : \xi_1^2 + (\xi_2 - \frac{1}{2})^2 < a^2\}$, $0 < a < \frac{1}{2}$. Let us denote $G_\varepsilon^j = \{x \in \Omega : \varepsilon^{-1}(x_1 - j, x_2) \in G\}$, $j \in \mathbb{Z}$, $G_\varepsilon = \bigcup\limits_j G_\varepsilon^j$, $\Gamma_\varepsilon = \partial G_\varepsilon$. We define the domain Ω_ε as $\Omega \setminus \overline{G_\varepsilon}$ (see Fig. 1).



We consider the following problem:

$$\begin{cases} \dfrac{\partial u_\varepsilon}{\partial t} - \nu \Delta u_\varepsilon + (u_\varepsilon, \nabla) u_\varepsilon = g(x), & x \in \Omega_\varepsilon, t > 0, \\ (\nabla, u_\varepsilon) = 0, & x \in \Omega_\varepsilon, t > 0, \\ \nu \dfrac{\partial u_\varepsilon}{\partial n} = 0, & x \in \Gamma_4, t > 0, \\ u_\varepsilon = 0, & x \in \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup \Gamma_\varepsilon, t > 0, \\ u_\varepsilon = U(x), & x \in \Omega_\varepsilon, t = 0. \end{cases} \tag{1}$$

Here $u_\varepsilon = u_\varepsilon(x,t) = (u_\varepsilon^1, u_\varepsilon^2)$, $g = g\,(x_1, x_2) = (g^1, g^2), g_j \in L_2(\Omega)$, n is the outer normal vector to the boundary and $\nu > 0$. Also consider the problem

$$\begin{cases} \dfrac{\partial u_0}{\partial t} - \nu\Delta u_0 + (u_0, \nabla)u_0 = g(x), & x \in \Omega, t > 0, \\ (\nabla, u_0) = 0, & x \in \Omega, t > 0, \\ u_0 = 0, & x \in \Gamma, t > 0, \\ u_0 = U(x), & x \in \Omega, t = 0. \end{cases} \tag{2}$$

We continue the solutions of the problem (1) by zero inside the pores.

Theorem. *Let u_ε be a solution to problem (1), u_0 be a solution to problem (2). Then, as the small parameter ε tends to zero, we have: $u_\varepsilon \to u_0$ strongly in $L_2((0,T), L_2(\Omega))$.*