Group analysis of the one-dimensional kinetic equation and the closure problem for the momentum system

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The problem of obtaining the continuous medium equations from kinetic equations based on group methods of analysis is discussed. The main idea is to calculate the symmetry group of the kinetic equation, transfer its action to moment variables, find the invariants of this group as functions of moment variables and, by setting invariant relations between them, cut and close the infinite system of moment equations, obtaining a finite system of continuous medium equations.

This idea is realized [PB21, PB18] on the simplest one-dimensional kinetic equation

$$f_t + cf_x + (Ff)_c = 0. \tag{1}$$

Group analysis of equation (1) is carried out in the class of diffeomorphisms of the space of all variables t, x, c, f (and also F, in the case of an equivalence group), which satisfy the following three conditions:

- the condition of invariance under these transformations of the relations

  $$dx = c\,dt, \qquad dc = F\,dt, \tag{2}$$

  which expresses the conservation of the relation between the physical quantities (t, x, c, F);
- the condition of invariance of the family of lines

  $$dx = dt = 0, \tag{3}$$

  necessary to preserve the physical meaning of the moment variables;
- condition of invariance under changes of variables of the quantity

  $$(1 + c\theta_x + F\theta_c)f(t, x, c)dxdc, \tag{4}$$

  on any surface $t = \theta(x, c)$, which expresses the independence of the number of particles from the choice of the coordinate system.

It was established that the group of point transformations of the space of variables (t, x, c, f, F) leaving relations (2), (3) and quantity (4) invariant coincides with the group of diffeomorphisms of the space of variables (t, x) and the transformations of the remaining variables generated by them; the equivalence group of equation (1) coincides with this group.

A group classification of equations (1) in the specified class of transformations was carried out. For the obtained symmetry groups, the action of these groups on moment variables was calculated and invariants were found.

In the case of $F = 0$, the differential invariant led to the system $\rho_t + (\rho u)_x = 0, \qquad u_t + uu_x = 0$, which is well known as the equations of "pressureless hydrodynamics". In this case, each solution $(\rho(t, x), u(t, x))$ with initial values $\rho_0(x)$ and $u_0(x)$ corresponds to the distribution $f(t, x, c) = \rho_0(x - ct)\delta(c - u_0(x - ct))$.

For equations with three-dimensional (submaximal in dimension) symmetry groups, the original statement of the problem was transformed into the following: we must first solve the system from equation (1), on the one hand, and from the equations representing the condition of the invariant expression of f through the first two moment variables from the other, and construct after for these f equations of a continuous medium that relate precisely these two moment variables.

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[PB18] K.S. Platonova and A.V. Borovskih, *Group analysis of the one-dimensional Boltzmann equation: III. Condition for the moment quantities to be physically meaningful*, Theoretical and Mathematical Physics **195**:3 (2018), pp. 886–915.

[PB21] K.S. Platonova and A.V. Borovskih, *Group analysis of the one-dimensional Boltzmann equation. Invariants and the problem of moment system closure*, Theoretical and Mathematical Physics **208**:3 (2021), pp. 1165–1181.