The Lagrange Identity and Dynamics in a Potential Jacobi Field

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A Jacobi field is a potential force field whose potential is a homogeneous function of degree -2. The problem of the motion of a particle in such a field admits an additional integral quadratic in velocities. It can be used to reduce the number of degrees of freedom and to pass to the study of a reduced system with spherical configuration space. These results are extended to the more general case of the motion of a particle in spaces of constant curvature. An analysis is made of particle motion on a cone whose vertex coincides with the singular point of the Jacobi potential. A lower estimate of the distance from the moving particle to the vertex of the cone is given. This approach is also applicable to a more general case where the charged particle is additionally located in the magnetic field of a monopole. A billiard inside the cone with a particle bouncing elastically off its boundary is considered.