Solutions Describing Particle Trajectories in the Field of Soliton-Like Wave Structures in a Fluid Beneath an Ice Cover: Approximation of Flows on a Central Manifold by Integrable Normal Forms

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A fluid layer of finite depth is described by Euler's equations governing the motions of the ideal fluid (water). The ice is assumed to be solid and it freely floats on the water surface. It is modeled by a geometrically nonlinear elastic Kirchhoff-Love plate. The trajectories of liquid particles under the ice cover are found in the field of different nonlinear surface traveling waves of small, but finite amplitude. These waves are: the classical solitary wave of depression, existing on the water-ice interface when the initial stress in the ice cover is large enough, the generalized solitary wave tending to the periodic wave at infinity [1], the envelope solitary wave and the so-called dark soliton [2]. The last two waves indicate the focusing or defocusing of nonlinear carier surface wave, the generalized solitary wave consists of solitary wave core and periodic asymptotic wave at spacial infinity, moreover for the algebraically small amplitude of the wave core of the generalized solitary wave, the amplitude of the mentioned above periodic wave is exponentially small. The consideration is based on explicit asymptotic expressions for solutions describing the mentioned wave structures on the water-ice interface, as well as asymptotic solutions for the velocity field in the liquid column corresponding to these waves. These expressions are given by the solution of the finite dimensional reduced equations, which are obtained by projection of the infinite dimensional differential equations, modelling the system in question, to the central manifold. The reduced equations are approximated to any algebraic order in the wave amplitude by integrable normal form equations, whose solutions may be found explicitly.

References

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