Central Configurations by Computer Algebra

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The n-body problem of celestial mechanics has simple motions, the relative equilibria. The configuration of the bodies in such a motion is called central.

In 1996 I proved by using polynomial elimination on computer that there are only 3 types of symmetric noncollinear central configurations of 4 equal masses. My preceding paper had proved the symmetry of the central configurations with 4 equal masses.

In 2019 another computer assisted method, based on interval arithmetic, reproved these two results and the corresponding ones for 5, 6 and 7 bodies with equal masses [1].

In a central configuration with rational masses the ratio of two mutual distances is an algebraic number. In my problem one of these ratios is a root of an irreducible polynomial of degree 37 with integer coefficents. The first coefficent is 39858075. Only one root is relevant.

A main inconvenience with this method is the following. The polynomial cannot be further simplified. If the power law of force is changed, the set of solutions remains qualitatively unchanged, while the complexity of the polynomial drastically increases with the complexity of the power (which should be a rational number in this method). If the law of attraction is in fourth power of the inverse distance instead of second power, the degree is 109, the first coefficient is -51414728063385600, but there is again only one relevant root. In contrast, the first power of the inverse distance gives very simple polynomials. We do not know how to reduce the study to the case of this simpler power law. This method produces too complicated polynomials with many useless roots.

The above two computer assisted methods are less efficient when dealing with equations with parameters. In our question the important parameters are the masses of the 4 bodies. The thesis of Leandro (2003) under the direction of Moeckel gave a simple picture. A recent publication [2] tries another way to get the solution. But still the paper is 33 pages long, only for the simplest case.

I will show small ideas to improve the method, big polynomials characterizing some bifurcation sets, and small results from this material.

References

[1] Moczurad M., Zgliczyński P., Central configurations in planar n-body problem with equal masses for $n = 5, 6, 7$, *Celest. Mech. Dyn. Astr.*, 2019, vol. 131, no. 10, Art. 46, 28 pp.

[2] Roberts G. E., On kite central configurations, *Nonlinearity*, 2025, vol. 38, no. 7, Paper No. 075001, 33 pp.