On Orbital Stability of Periodic Solutions of Hamiltonian System with Two Degrees of Freedom in Resonant Cases of Degeneration

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Periodic solutions of autonomous Hamiltonian systems usually form a family depending on one or more parameters. In the general case, the period of these solutions continuously depends on these parameters. The above circumstance leads to instability of periodic solutions in the sense of Lyapunov. On the other hand, the periodic solutions which are unstable in the sense of Lyapunov can be orbitally stable. The problem of orbital stability is of the great interest both from a general theoretical point of view and for applications in rigid body dynamics, celestial mechanics and satellite dynamics. Modern methods of the theory of dynamical systems, stability theory and the qualitative theory of differential equations allow to obtain a solution to this problem in a strictly nonlinear sense.

The most complete and rigorous conclusions on the orbital stability of periodic solutions have been obtained for autonomous Hamiltonian systems with two degrees of freedom. In particular, in cases, where the stability problem can be solved by terms of order no higher than fourth in the expansion of the Hamiltonian function in the neighborhood of a periodic solution, rigorous orbital stability criteria have been formulated and proven in [1–3] based on KAM theory methods. At present, in the problem of orbital stability of periodic solutions of an autonomous Hamiltonian system with two degrees of freedom, only special degenerate cases remain unexplored. In these cases, the analysis of fourth-order terms is insufficient to obtain rigorous conclusions about orbital stability and it is necessary to perform stability study by taking into account higher-order terms. In particular, such a degeneration arises in resonance cases.

In this work, we study the problem of orbital stability of periodic solutions of an autonomous Hamiltonian system with two degrees of freedom in previously unexplored cases of third, fourth and sixth order resonances, when, in order to obtain rigorous conclusions, it is necessary to take into account the terms of the sixth order in the Hamiltonian expansion in the neighborhood of the periodic solution. We obtain sufficient conditions of orbital stability and instability of the periodic solutions in a form of inequalities with respect to coefficients of Hamiltonian normal form calculated up to terms of sixth

order. We apply the results of this study in the problem of orbital stability of periodic motions of heavy rigid body, whose principal moments of inertia satisfy the relation A=C=4B.

The work is supported by the grant of the Russian Science Foundation (project 24-11-00162, https://rscf.ru/en/project/24-11-00162/) and was carried out at the Moscow Aviation Institute (National Research University).

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