On Robustly Chaotic Attractors in the Generalized Kuramoto Model

Alexey Kazakov

HSE University, Nizhny Novgorod, Russia

In this talk we consider the generalized Kuramoto model

$$\dot{\phi}_j = \omega + \frac{1}{N} \sum_{i=1}^{N} g(\phi_i - \phi_j), \quad j = 1, \ldots, N, \tag{1}$$

where $\phi_j \in S^1$ is the phase of the j-th oscillator, N is the number of the oscillators, ω is the common frequency, and the 2π-periodic function $g(\phi)$ describes the coupling.

We show that for $N = 4$ this system can demonstrate, in certain regions of the parameter space, robustly chaotic attractors, with no stability windows. We give a rigorous proof of the existence of these regions and provide explicit conditions on the coupling function $g(\phi)$ which allow one to find them for arbitrary 2π-periodic function $g(\phi)$. We also show that the presence of the 4th harmonic is necessary for the existence of the robustly chaotic attractors.

The theoretical results are applied to an example of the coupling function with four Fourier modes: $g(\phi) = \sum_{k=1}^{4} A_k \sin(k\,\phi + \xi_k)$. In this case we find the regions with robustly and non-robustly chaotic attractors numerically. It is worth noting that the robustly chaotic attractors that we found remain chaotic under small time-dependent perturbations (periodic, quasiperiodic), and any network of weakly coupled, identical systems with attractors of this type is also robustly chaotic.

This is a joint work with E. Karatetskaia, K. Safonov, and D. Turaev.

This work was supported by the RSF grant No. 25-11-20069.