Integrable Magnetic Flows on n-dimensional Spheres and Nonholonomic Mechanics

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We introduce and study the Chaplygin systems with gyroscopic forces. We put a special emphasis on the important subclass of such systems with magnetic forces. In a reduction, we construct Hamiltonian magnetic systems on spheres S^n. Recently, we provide a Lax representation of the equations of motion and prove complete integrability of those systems for any n. The integrability is provided via first integrals of degree one and two.

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