

# On Integrability of Magnetic Geodesic Flows on 2-Surfaces at Different Energy Levels

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We consider geodesic flows in a magnetic field on 2-surfaces. In contrast with the standard geodesic flows (with zero magnetic field), dynamics of magnetic ones at different energy levels is different (e. g., see [1]). This is why an integrability of such flows at all energy levels simultaneously is a rather rare phenomena (e. g., see [2–4] where this problem has been studied on the 2-torus).

Due to this, it is very natural to study the question about integrability of such flows at a fixed energy level only. The case of an additional quadratic in momenta first integral was investigated in [5]. As shown in [5], this problem can be reduced to a certain semi-Hamiltonian system of PDEs [6]. In the talk we will discuss various methods of constructing smooth and analytical solutions to this system on the 2-torus [7, 8]. We also consider the case of an additional rational in momenta first integral and construct a family of local explicit integrable examples [9].

## References

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