Nonclassical Discontinuities for Nonstrictly Hyperbolic Conservation Laws

Anna Chugainova

Steklov Mathematical Institute of RAS, Moscow, Russia

Nonclassical discontinuities and their role in solving Riemann problem are studied. Solutions to a system of two hyperbolic equations representing conservation laws are investigated. On the one hand, this system of equations makes it possible to demonstrate the non-standard solutions to the Riemann problem, on the other hand, this system of equations describes longitudinal-torsional waves in elastic rods. We use the traveling wave criterion for admissibility of shocks as the additional jump condition. If the dissipation parameters included in each of the equations of the system are different, then there are undercompressive and overcompressive waves.

We have numerically studied the asymptotics of the main types of solutions to the Riemann problem for the system of equations describing nonlinear longitudinal–torsional waves in viscoelastic media [1]. The study has shown that the asymptotics of nonstationary solutions of the Riemann problem may contain undercompressive shock (nonclassical discontinuities). The solutions with a undercompressive shock are formed for a certain relation between the dissipation parameters appearing in the equations. If two dissipation parameters are identical (or their ratio is close to unity), then the asymptotics of the solution corresponds to the self-similar solution and does not contain undercompressive shocks. We have shown that, for different ratios of dissipation parameters, one can obtain different solutions to the Riemann problem for the same initial data.

References

[1] Chugainova A. P., Riemann problem for longitudinal–torsional waves in nonlinear elastic rods, *Z. Angew. Math. Phys*, 2024, vol. 75, no. 106.