Method of Quasisolutions Applied to R. Peierls' Theory of Thermal Conductivity

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In his seminal 1929 paper [1], R. Peierls' described thermal conductivity in solids and provided a heuristic derivation of Fourier's law from microscopic dynamics of particles. By analyzing a lattice of nonlinearly coupled oscillators, he demonstrated that, in the limit of an infinite number of oscillators and vanishing nonlinearity, the energy distribution across Fourier modes satisfies a specific nonlinear kinetic equation. Although this result had motivated considerable efforts by mathematicians and mathematical physicists, a fully rigorous derivation remains an open problem.

Inspired by Peierls' kinetic theory, the field of wave turbulence theory emerged in the second half of the 20th century, focusing on the statistical properties of weakly nonlinear wave systems. Recent years have seen substantial progress in its rigorous justification.

In this talk, I will discuss ongoing work that revisits a setting similar to Peierls' original model, employing techniques from recent advances in wave turbulence. Starting from a d-dimensional lattice of oscillators with weak nonlinear coupling and stochastic perturbation, I will present the derivation of a kinetic equation for the energy spectrum of a "quasisolution" — an approximation that accurately captures the expected behavior of the true solution. I will provide an overview of the key steps in this derivation and discuss the technical challenges we have encountered in the process.

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References

[1] Peierls' R., Zur kinetischen Theorie der Wärmeleitung in Kristallen, in *Annalen der Physik*, vol. 395, no. 8, Weinheim: Wiley-VCH GmbH, 1929, pp. 1055–1101.