# Sparkling Saddle Loops of Vector Fields on Surfaces and Related Issues

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If an orientation-preserving separatrix loop of a hyperbolic saddle of a vector field on a two-dimensional surface is accumulated by a separatrix of another saddle, sparkling saddle connections appear when the loop is unfolded in a one-parameter family of vector fields. If it is the "free" unstable separatrix of the same saddle that accumulates to the loop, we get sparkling saddle loops instead. This can occur only on surfaces different from $S^2$ and $\mathbb{RP}^2$, of course.

The corresponding generic semi-local bifurcations differ dramatically depending on whether the path along the free separatrix preserves the local orientation or reverses it.

In the first case, the parameter values for which the vector field has a saddle loop are the endpoints of the gaps in a Cantor set. A thin tubular neighborhood of the unstable manifold of our saddle is homeomorphic to a torus with a disk removed. By cutting out this neighborhood and gluing a disk to it, we obtain a family of vector fields on the torus. The remaining points of the aforementioned Cantor set in the parameter space correspond to the Cherry flows on this torus: the Poincaré map has irrational rotation number, and hence the flow has nontrivial recurrent trajectories.

In the case when the path along the "free" unstable separatrix reverses the orientation, there are only two alternating convergent sequences that correspond to separatrix loops involving different unstable separatrices. Assuming the saddle is dissipative, flip-cycles are born and destroyed when these loops are unfolded, and the two sequences split the one-sided neighborhood of the critical parameter value into intervals where either one or two such cycles are present.

The configuration in which an orientation-reversing saddle loop of a dissipative saddle is accumulated by its free unstable separatrix does not appear in generic $C^1$ one-parameter families of vector fields.