Orbital Stability of Overcompressed Discontinuities of a Hyperbolic 2×2 System of Conservation Laws

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This paper is devoted to the study of the orbital stability of overcompressed discontinuities (shocks) in a hyperbolic system of two conservation laws. The system describes longitudinal-torsional waves of small amplitude propagating in one direction in a nonlinear elastic rod and was originally derived in the works of [1,2].

The standard method of viscous regularization is applied to the system. This regularization leads to the appearance of non-classical discontinuities — specifically, overcompressed shocks.

In the phase plane, an overcompressed shock is represented by a one-parameter family of integral curves (viscous profiles) connecting two nodal points. Any of these viscous profiles can represent the structure of the discontinuity. In this work the study of the stability of these viscous profiles is studied. The stability analysis is performed using the Evans function technique, which demonstrates that these viscous profiles are orbitally stable.

Furthermore, the Riemann problem is numerically solved for initial data corresponding to the overcompressed discontinuity. The computational results show that only one specific viscous profile from the continuous family emerges as the long-time asymptotics of the solution. The other profiles may also arise in non-stationary calculations, but only for specific initial conditions.

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References

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