A Semi-Analytical Approach to Secular Effects in the Motion of Earth's Quasi-Satellites

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A quasi-satellite of a planet is a small body that stays in the planet's vicinity for long intervals at heliocentric distances much smaller than the planet–Sun distance, yet always far outside the planet's Hill sphere [1]. Quasi-satellite motion (later will be denoted as QS motion) corresponds to the 1:1 mean-motion resonance of the object and the planet (Fig. 1). For Earth, several such objects are currently cataloged (e. g., Cardea, 2006 FV35, 2013 LX28, 2014 OL339, Kamo'oalewa, 2020 PP1, 2022 YG, 2023 FW13).

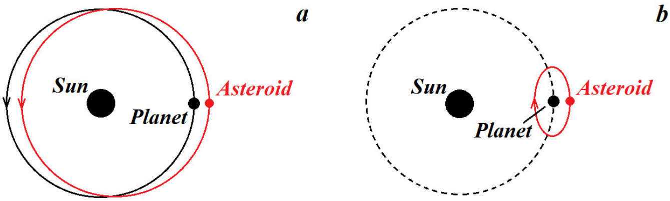

Figure 1. Orbital motion of a planet and its quasi-satellite: Panel **a** motion in a heliocentric reference frame whose axes keep a fixed orientation in inertial space; the planet moves around the Sun on a circular orbit, while the quasi-satellite follows an elliptical one; Panel **b** motion of the planet and the quasi-satellite in a rotating coordinate frame that keeps the Sun–planet direction unchanged [2].

The relative position of a quasi-satellite and Earth during their orbital motion is conveniently tracked by the difference of their mean longitudes $\varphi = \lambda - \lambda_3$, we will refer to it as the resonant phase considering that we are describing resonant motion. Due to the large distance from the planet, quasi-satellite's motion can be treated as a weakly perturbed heliocentric motion, so perturbation techniques are suitable for following how φ evolves over time (e. g., [2,3]). Averaging over the orbital motion of the quasi-satellite and those planets that exert the main influence leads to a compact evolution equation for φ:

$$3\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} + \mu\frac{\partial W}{\partial \varphi} = 0, \qquad (1)$$

where μ is the Earth-to-Sun mass ratio, and W is the disturbing function that characterizes how the gravitational influences of Earth and the other planets to the quasi-satellite's motion. Fig. 2 shows examples of graphs illustrating the dependence of the value of the disturbing function W on the resonant phase φ for real quasi-satellites of the Earth.

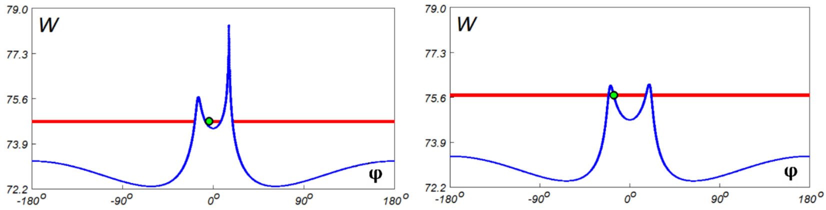

Figure 2. Averaged disturbing function for asteroids (164207) Cardea (left) and 2023 FW13 (right). The red lines show the value of energy integral of the evolution equation (1) that matches the observed motion of the asteroids. The dots mark the current values of the resonant phase.

Over time, an object's resonant regime can change, for instance, from QS regime to HS regime (corresponds to movement on horseshoe like orbit in rotating frame) or to combined QS+HS regime. Such changes may occur in a deterministic or in a probabilistic manner. Type of transition depends on the values of the problem's first integrals. Formulas for the probabilities of transitions between resonant states are given in [4], for the present case they can be written as follows:

$$P_{QS,HS} = \frac{\widehat{\Theta}_{QS,HS}}{\widehat{\Theta}_{QS} + \widehat{\Theta}_{HS} + \widehat{\Theta}}, \quad P_{QS+HS} = 1 - P_{QS} - P_{HS},$$

where $\widehat{\Theta}_{QS,HS} = \max\bigl(\Theta_{QS,HS},\, 0\bigr)$, $\widehat{\Theta} = \max\bigl(-\Theta_{QS} - \Theta_{HS},\, 0\bigr)$. The quantities $\widehat{\Theta}_{QS,HS}$ represent the rates of change of the areas in the phase plane of equation (1) that are occupied by trajectories corresponding to the QS and HS regimes, respectively.

Described procedure was applied to Earth's quasi-satellites. Different scenarios of their evolution were established.

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References

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