On Discrete Lorenz-like Attractors of Three-dimensional Maps

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We observe main elements of the theory of discrete attractors of Lorenz type in the case of three-dimensional maps. We select, first of all, two different types of such attractors: the so-called standard discrete Lorenz attractors, which, as shown in [1–4], appears in nonlinear maps of general type, as well as symmetric discrete Lorenz attractors that are characteristic for maps with the axial symmetry [5], (i. e. the symmetry $x \to -x, y \to -y, z \to z$ which is the same as the symmetry in the flow Lorenz model). For various types of discrete Lorenz attractors, we describe their basic geometric and dynamical properties, and also present main phenomenological bifurcation scenarios in which they arise. We also consider specific examples of discrete Lorenz attractors of various types in three-dimensional quadratic maps, such as three-dimensional Hénon maps and quadratic maps with axial symmetry and constant Jacobian. For the latter, their normal forms are presented, i. e. universal maps, to which any map from a given class can be reduced by means of linear coordinate transformations. For these normal forms we study bifurcations leading to symmetric discrete Lorenz-like attractors of different types, including the so-called twisted Lorenz attractors [5].

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