Analysis of the Dynamics of the Controlled Motion of a Three-Link Wheeled Mobile Robot within the Framework of Different Friction Models

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The problem of the controlled motion of a three-link wheeled mobile robot is addressed. It is assumed that the wheeled vehicle consists of three equal platforms (links) connected to each other by means of cylindrical joints, with wheel pairs rigidly fastened to the platforms. The points of attachment of the platforms to each other lie on straight lines that pass through the centers of mass of the platforms and are perpendicular to the axes of the wheel pairs. The center of mass of each platform lies on a straight line that passes through the point where the platforms are connected to each other and the wheel pair is fastened. It is assumed that the system moves on a horizontal plane and that its motion is due to periodic oscillations of the platforms relative to each other. The turning angles of the lateral platforms relative to the central platform are prescribed periodic functions of time (control functions).

In this paper we consider the dynamics of a three-link mobile robot within the framework of three different models of motion, and compare them with experimental results. The first of the models is the kinematic nonholonomic model of motion. Nonholonomic constraints imposed on the system correspond to the no-slip constraint at the points of contact of the wheels with the plane of motion. It is shown that in this case there exist ranges of values of control functions (special configurations) in which the constraint reactions increase without bound regardless of the type of these controls. As the system passes these configurations, the constraint reactions generally begin to increase without bound, so that the nonholonomic model becomes inapplicable.

The second model is the stick-slip hybrid model induced by Coulomb friction, which is developed in [1] for the three-link mobile robot. In this model it is assumed that the lateral slipping (along the axis of the wheel pair) begins at the moment when the reaction forces go beyond the limits of the cone of friction defined by the law of dry Coulomb friction. In this case, a reverse transition to the nonholonomic rolling is possible at the moment when the velocity of slipping becomes zero.

The third model is the model of viscous friction, in which we assume that the motion occurs with lateral slipping, and that the force of viscous friction

that is proportional to the velocity of lateral slipping acts at the points of contact. In this case it is necessary to take account of the degrees of freedom related to the rotation of the wheels, in contrast to the nonholonomic model in which the wheel pairs can be replaced with a knife edge (skate) located at its center of mass and prohibiting sliding in the transverse direction (relative to the plane of the wheels) [2] . In this model, we investigate the influence of the friction force on the dynamics under the assumption that forces of viscous friction with a Rayleigh function proportional to the square of the velocity of lateral slipping act at the points of contact of the wheels with the plane.

To estimate the adequacy of the above-mentioned models of motion and to determine the scope of their applicability, a series of experiments with a prototype of the three-link wheeled robot with periodic controls were carried out. For "slow" controls (a period larger than some period defined experimentally) the experimental results are in good qualitative and quantitative agreement with the nonholonomic model and the model of viscous friction. As the period of control functions decreases, one can experimentally observe a lateral slipping, for which the model of viscous friction also provides an adequate qualitative and quantitative description.

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References

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[2] Borisov A. V., Kilin A. A., and Mamaev I. S., On the Hadamard – Hamel problem and the dynamics of wheeled vehicles, *Regul. Chaotic Dyn.*, 2015, vol. 20, no. 6, pp. 752–766.