On Bifurcations Leading to the Instant Creation of Hyperchaotic Attractors with Two and Three Positive Lyapunov Exponents

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It is well kown that chaos in the parabola map $\bar{x} = 1 - ax^2$ appears via the cascade of period-doubling bifurcations [1]. In this report we present results of the study of the Kopel duopoly model [2]:

$$\bar{x} = (1-\rho)x + \rho\mu y(1-y), \quad \bar{y} = (1-\rho)y + \rho\mu x(1-x),$$

where x and y are output quantities of each firm, parameter μ measures the intensity of the effect that one firm's action has on the other firm, and ρ is the adjustment coefficient of the adaptive expectations of the firm. For this case our numerical experiments show that an infinite sequence of the degenerate period-doubling bifurcations (when both multipliers are equal to -1 at the bifurcation moment) accumulates to the so-called "double Feigenbaum point" [3] belonging to the boundary between periodic and hyperchaotic dynamics with two positive Lyapunov exponents.

We also extend our results to the case of hyperchaotic dynamics with three positive Lyapunov exponents by the example of the Kaneko map [4]:

$$\bar{x} = Ax + (1-A)(1-Dy^2), \quad \bar{y} = z, \quad \bar{z} = x.$$

This map depends on two parameters A and D. Along $A = 0$ at each period-doubling bifurcation we have three multipliers on the unit circle simultaneously: $\lambda_1 = -1$ and $\lambda_{2,3} = e^{\pm i\pi/3}$, i. e., a cascade of degenerate period-doubling bifurcations occurs. This cascade results in a hyperchaotic attractor with three positive Lyapunov exponents. We continue the region with such attractor on the (A, D)-plane.

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References

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