Statistical Properties of Extreme Events in the Ring of FitzHugh – Nagumo Neurons

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Extreme events in society and nature, such as pandemic outbreaks, abnormal waves, or structural failures, can have catastrophic consequences. An extreme event is an event in which the value under consideration shows significant deviations from the average value, several times higher than the standard deviation from the average value, and with an outbreak of activity leads to a change in the normal functioning of the system. This phenomenon is also observed with the extreme work of brain neurons during epileptic seizures [1]. Therefore, this kind of research is important and relevant to study extreme events in neural networks.

The most common biologically significant and simple neuron model is the FitzHugh – Nagumo oscillator [2]. A network consisting of this neuron model can show how the brain works, and also helps simplify the study of the effects of various kinds of phenomena and external influences on the dynamics of neurons. In this study, Poisson impulses and Lévy noise were used as external influences, which make it possible to control processes in neural systems.

A system of coupled neurons was used to study extreme events in the ring of FitzHugh – Nagumo neurons:

$$\begin{aligned} \varepsilon \dot{x}_i &= x_i \left(1 - \frac{u^3}{3}\right) - y_i + F(t), \\ \dot{y}_i &= x_i + a, \end{aligned} \qquad (1)$$

where x_i and y_i are dynamic variables that describe the temporal dynamics of the activator (fast variable) and the inhibitor (slow variable), respectively; $i = 1, 2, \ldots, N$ is the element number, and $N = 10$ is the total number of oscillators in the network.

$F(t)$ is responsible for the external effect on the network of neurons. In the case when Poisson pulses are used as an external influence, $F(t) = A\xi^f(t)$, where f is the frequency parameter, and A in (1) is the amplitude of the signal. Poisson pulses are characterized by an independent time between the moments of pulse delivery and have an exponential distribution. If Lévy noise is used as an external influence, then $F(t) = \eta_i^{\alpha^L, \beta^L}(t, \sigma^L)$, where the parameter α is the stability index showing the rate of narrowing of the tails of the distribution. The parameter $\beta^L \in [-1; 1]$ defines the skewness of the distribution, and σ defines the width of the distribution. Couplings between neurons are subject to periodic boundary conditions ($i + kN = i$ for any integer k).

Extreme events exceeding the three standard deviations have a greatest range of probability of observation. When the coupling strength is attractive, it affects the behavior of the system, bringing the distribution of spike amplitude closer to normal in some optimal range of coupling strengths. With a high repulsive coupling, leakage is observed in areas with a lower probability, while the entire parameter plane has a higher probability. Statistical characteristics of extreme events under external influence were also considered. The analysis of the relationship of statistical characteristics with the behavior of the system is carried out.

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References

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[2] FitzHugh R., Mathematical models of threshold phenomena in the nerve membrane, *The bulletin of mathematical biophysics*, 1955, vol. 17, no. 4, pp. 257–278.