Morsifications of Semihomogeneous Functions with Any Possible Number of Critical Points

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A deformation of an analytic function is called a *morsification* if it has only nondegenerate critical points. It is well-known that any morsification of a germ of an analytic function has the same number of complex critical points, known as a Milnor number (or multiplicity) of a germ. By contrast, different real morsifications of a real analytic function can have different amounts of real critical points. Constructions of morsifications with controlled number and locations of critical points were considered by various authors (e. g. [1,3–8]). Such morsifications have applications in various areas of singularity theory, such as the Picard-Lefschetz theory (where a specific type of morsification can be used for monodromy calculation [1,2]) and the theory of Lagrangian singularities [4]. V.Vassiliev has constructed morsifications with all possible numbers of real critical points for all simple real singularities [4] and some real singularities of modality one [5]. Here we would like to present explicit constructions of morsifications with all topologically permissible numbers of critical points for real semihomogeneous function germs of two variables with real components of zero level set.

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