Application of Poisson Learning for the Classification of Solutions of Hamilton Systems

Constantin Ruchkin

"CR Consult", Donetsk, Russia

In this work, we consider Hamiltonian systems and investigate their behavior using modern machine learning techniques — specifically, graph-based semi-supervised Poisson learning. The general objective of this study is to develop an algorithmic framework that enables automatic classification of Hamiltonian system solutions in real time. The analytical foundation of the research is based on the Poincaré section method, which provides an informative two-dimensional projection of the system's phase space. The structure of the resulting phase space consists of multiple trajectories represented by periodic curves and/or chaotic point clouds. The trajectories are constructed through numerical integration of differential equations with given initial conditions in real time. However, this process is time-limited due to accumulated numerical error, which also poses a major challenge for full automation of the classification procedure. As a result, after a fixed simulation period, the two-dimensional Poincaré sections yield trajectory flows with distinct properties that must be classified into separate categories within a limited time window.

To address this problem — the classification of trajectory projections of phase-space flows on two-dimensional manifolds — we propose the use of a graph-based semi-supervised Poisson learning (PL) approach. Graph-based semi-supervised learning methods represent a modern machine learning framework for solving data classification problems when the use of fully supervised methods is impractical or impossible. The goal of this approach is to achieve high classification performance with a minimal number of labeled data samples. Poisson learning assumes that labels are known for a subset of data points, and these labels are then propagated to the remaining nodes using a harmonic function defined by the Poisson equation. This method has a variational formulation leading to the minimization of the Dirichlet energy functional. The corresponding optimization problem can be solved iteratively, for instance, using the Jacobi method. In the context of semi-supervised machine learning, such an iterative propagation process can be interpreted as a random walk on graphs. In this work, we also introduce a modified iterative random-walk solution designed to accelerate the learning and classification process. Thus, the proposed approach enables real-time automatic classification of Hamiltonian system solutions [1] while accounting for numerical accumulation errors.

References

[1] Ruchkin C., The General Conception of the Intellectual Investigation of the Regular and Chaotic Behavior of the Dynamical System Hamiltonian Structure, in *Applied Non-Linear Dynamical Systems. Springer Proceedings in Mathematics and Statistics*, vol. 93, Cham: Springer, 2014, pp. 245–254.