On Different Types of Hyperbolic Chaotic Sets Appearing as a Result of the Perturbations of Anosov Map on a 2D Torus

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The report is devoted to the study of a two-parameter diffeomorphism obtained by perturbing the Anosov map using the Möbius map on a two-dimensional torus. The Möbius map depends on two parameters which are responsible for the dissipativity and shift of coordinates [1]. In the parameter space of the studied diffeomorphism, three regions were identified depending on the structure of the non-wandering set: (1) the non-wandering set covers the entire torus; (2) the non-wandering set consists of a fixed point and a one-dimensional hyperbolic set; (3) the non-wandering set consists of stable and completely unstable fixed points and a zero-dimensional chaotic set. The second region is divided into two subregions: (2.1) the fixed point is a repeller, and the one-dimensional hyperbolic set is an attractor; (2.2) the fixed point is an attractor, and the one-dimensional hyperbolic set is a repeller. To study the hyperbolicity of the map and the structure of the non-wandering set numerically, we use the calculation of Lyapunov exponents and angles between tangent subspaces [2, 3]. The transition from the first region to the second was described in detail in [4], so this work focuses on the study of the third region. In this parameter region, a zero-dimensional chaotic set is generated, which is neither an attractor nor a repeller, so in order to approximate this invariant set, the method described in the work [5] is used. The hyperbolicity of all the described sets is checked, and the bifurcations that occur during transitions between different regions are described.

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References

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