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Eigenvalue distribution for some singular spectral problems

Let $\mathbf{A}$ be a positive self-adjoint operator and $\mathbf{V}$ be a quadratic form perturbation so that the domain of $\mathbf{A} + \mathbf{V}$ may differ from domain of $\mathbf{A}$. If these domains, in fact, differ, we call this situation a *singular perturbation*. We consider two spectral problems.

1. Estimates and asymptotics of the number of negative eigenvalues of $\mathbf{A} + t\mathbf{V}$ as $t \rightarrow \infty$;
2. Estimates and asymptotics of eigenvalues of $(\mathbf{A} + \mathbf{V} + c)^{-m} - (\mathbf{A} + c)^{-m}$, provided this difference is compact.

Spectral analysis of these operators started in 1950-s and early 1960-s by M.Sh.Birman. We propose a modified approach to these problems, based upon recent results with E.Shargorodsky and G.Tashchiyan on spectral properties of Birman-Schwinger type operators with singular measures. As applications, we consider the cases of $\mathbf{A} + \mathbf{V}$ being the Laplacian with singular Robin boundary condition on the boundary, the Laplacian with δ or δ' -type potentials supported on a surface, and fractal potentials. If time allows, we present also some results on scattering theory for a singularly perturbed polyharmonic operator, a tribute to Dima Yafaev, who devoted one of his earliest papers to this topic.