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Singularities for planar webs through their abelian relations

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Abstract

Web geometry deals with families of foliations in general position. In the planar case locally in \mathbb{C}^2 or globally on $\mathbb{P}^2 := \mathbb{P}^2(\mathbb{C})$, a d -planar web $\mathcal{W}(d)$ is given by the generic family of integral curves of respectively an analytic or an algebraic differential equation of the first order $F(x, y, y') = 0$ with degree d . The main numerical invariant of such a $\mathcal{W}(d)$ is the rank of a local system defined outside the discriminant locus Δ associated with F . It is given by the *abelian relations* of $\mathcal{W}(d)$, that is special relations between the normals of its leaves. This rank is bounded by $\pi_d := \frac{1}{2}(d-1)(d-2)$ and the previous bound is optimal from Abel's addition theorem.

A d -planar web $\mathcal{W}(d)$ with $d \geq 3$ gives rise to a meromorphic connection (E, ∇) of rank π_d with poles on Δ which is not necessary integrable. Properties of this connection will be given, in particular a determinant formula for $(\det E, \det \nabla)$ will be presented which involves residues on Δ . In order to approach classification of *remarkable planar webs* $\mathcal{E}(d)$, that is with maximum rank but not linearizable, some other results will be also discussed as regularity and monodromy aspects.

Methods and tools used illustrate the rich interplay between differential geometry and algebraic geometry