ON MEROMORPHIC FUNCTIONS

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Abstract

By introducing a general growth scale, say, φ-order, we establish a general theory on the value distribution of a-points for any nonconstant meromorphic functions. We next prove the following deficiency relation and its variations :

Let f be nonconstant meromorphic in the complex plane $\mathbb{C}$ and of finite positive φ-order λ. If there is a positive number $\tau > 1$, such that

$$\log^+ \left(\varphi \left(r + \frac{1}{U(r,f)} \right) \right) = o\left(U(r,f)\right) \qquad \text{as } \; r \to +\infty,$$

here $U(r,f) = \varphi(r)^{\lambda(r)}$ with $\lambda(r)$ being a proximate φ-order for $T(r,f)$ the characteristic function of f, then

$$\sum_{a \in \widehat{\mathbb{C}}} \left\{ 1 - (1/\lambda) \lim \sup_{r \to +\infty} \frac{\overline{n}(r, f=a)\varphi(r)}{r\varphi'(r)T(r,f)} \right\}^+ \leq 2,$$

where $\overline{n}(r, f=a)$ is the number of distinct roots of the equation $f(z) = a$ for $|z| \leq r$, $\widehat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$ is the extended complex plane, the summation is over all extended complex values and the upper bound 2 can be reached. We also obtain a complete and precise description on $n(r, f=a)$ in terms of $\delta(a,f)$, $T(r,f)$ and λ for any meromorphic function $f(z)$ of perfectly regular order.

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