V. S. VLADIMIROV AND *p*-ADIC MATHEMATICAL PHYSICS

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CONFERENCE

Mathematical Physics. Vladimirov–90

Steklov Mathematical Institute – Moscow

VASILIY SERGEEVICH VLADIMIROV (9.01.1923 – 3.11.2012)

1. Relations to Serbian mathematicians
2. Some contributions to *p*-adic mathematical physics
3. Further developments of some Vladimirov investigations

1. I met Vladimirov for the first time in October 1982, during his visit to Novi Sad and Belgrade.
2. After that I met him many time, in particular in MIAN.
3. Since December 1987, foreign member of Voevodina Academy of Sciences and Arts (Novi Sad), and since May 1991 foreign member of Serbian Academy of Sciences and Arts (Belgrade).

- In Steklov Mathematical Institute.
- Conference on Generalized Functions (Novi Sad, 1996).

- Vladimirov had a high opinion on Serbian mathematicians Jovan Karamata (1902–1967) and Djuro Kurepa (1907–1993).
- Kurepa hypothesis on the left factorial

$$!n = \sum_{k=0}^{n-1} k!, \quad n = 1, 2, \ldots; \qquad (!n, n!) = 2, \quad n = 2, 3, \ldots$$

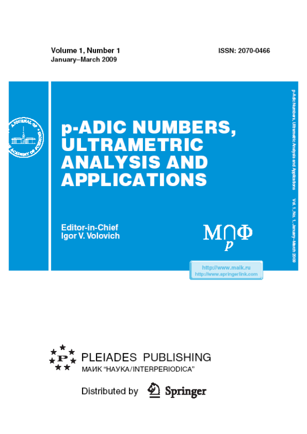
- Vladimirov paper “Left factorials, Bernoulli numbers and Kurepa hypothesis” in Publications de l’Institut Mathematique (Belgrade, 2002).
- Kurepa hypothesis in terms of p-adic numbers is

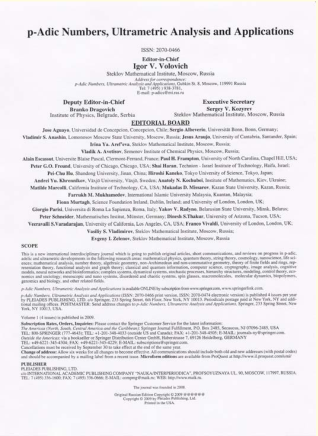
$$\sum_{k=0}^{\infty} k! = a_0 + a_1 p + a_2 p^2 + \cdots, \quad a_0 \neq 0 \text{ for } p \neq 2.$$

Some contributions to *p*-adic mathematical physics

- New *p*-adic and adelic string amplitudes
- Investigation of nonlinear nonlocal equations of motions for p-adic scalar strings
- One of founders of *p*-adic quantum mechanics
- Vladimirov *p*-adic pseudodifferential operator
- One of founders of international interdisciplinary journal *p-Adic Numbers, Ultrametric Analysis and Applications* and international meetings (conferences) on *p-adic mathematical physics*
- One of founders of *p-Adic Mathematical Physics*
- Contributions to *p*-adic analysis: to theory of *p*-adic generalized functions, Table of *p*-adic integrals

- International interdisciplinary journal “*p*-Adic Numbers, Ultrametric Analysis and Applications”





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The journal was founded in 2008.

- International conferences on *p*-Adic Mathematical Physics: Moscow 2003, Belgrade 2005, Moscow 2007, Grodno 2009, Bielefeld 2013.

2nd International Conference on p-Adic Mathematical Physics
Belgrade, 2005

- Vladimirov *p*-adic pseudodifferential operator

$$(D^{\alpha}\psi)(x) = \int_{\mathbb{Q}_p} |\xi|_p^{\alpha}\, \tilde{\psi}(\xi)\, \chi_p(\xi x)\, d\xi$$

- Investigated some basic properties.

- Vladimirov – one of founders of p-adic quantum mechanics
- According to Vladimirov and Volovich (Commun. Math. Phys., 1989) p-Adic Quantum Mechanics is a triple

$$(L_2(\mathbb{Q}_p), W_p(z), U_p(t))$$

where $L_2(\mathbb{Q}_p)$ is Hilbert space on $\mathbb{Q}_p$, $W_p(z)$ – unitary representation of the Heisenberg-Weyl group on $L_2(\mathbb{Q}_p)$, and $U_p(t)$ is a unitary representation of the evolution operator on $L_2(\mathbb{Q}_p)$.

- Harmonic oscillator and free particle were investigated.
- Feynman path integral was introduced.

Recall string amplitudes:

- standard crossing symmetric Veneziano amplitude

$$A_\infty(a,b) = g_\infty^2 \int_{\mathbb{R}} |x|_\infty^{a-1} \, |1-x|_\infty^{b-1} \, d_\infty x$$
$$= g_\infty^2 \frac{\zeta(1-a)}{\zeta(a)} \frac{\zeta(1-b)}{\zeta(b)} \frac{\zeta(1-c)}{\zeta(c)}$$

- p-adic crossing symmetric Veneziano amplitude

$$A_p(a,b) = g_p^2 \int_{\mathbb{Q}_p} |x|_p^{a-1} \, |1-x|_p^{b-1} \, d_p x$$
$$= g_p^2 \frac{1-p^{a-1}}{1-p^{-a}} \frac{1-p^{b-1}}{1-p^{-b}} \frac{1-p^{c-1}}{1-p^{-c}}$$

where $a = -s/2 - 1$ and $a, b, c \in \mathbb{C}$ and $a + b + c = 1$.

- Freund-Witten product formula for adelic strings

$$A(a,b) = A_\infty(a,b)\prod_p A_p(a,b) = g_\infty^2 \prod_p g_p^2 = const.$$

- Amplitude for real string $A_\infty(a,b)$, which is a special function, can be presented as product of inverse p-adic amplitudes, which are elementary functions.
- Vladimirov found an appropriate regularization for the above adelic infinite product.
- Vladimirov constructed new p-adic string amplitudes and adelic formulas
  1. in fields of algebraic numbers (1996)
  2. gamma and beta functions of one-class quadratic fields and application to Ramond-Neveu-Schwarz superstring

- There is an effective field description of scalar open and closed p-adic strings. The corresponding Lagrangians are very simple and exact. They describe not only four-point scattering amplitudes but also all higher (Koba-Nielsen) ones at the tree-level.
- The exact tree-level Lagrangian for effective scalar field φ which describes open p-adic string tachyon is

$$\mathcal{L}_p = \frac{m_p^D}{g_p^2} \frac{p^2}{p-1} \Big[-\frac{1}{2} \varphi \, p^{-\frac{\Box}{2m_p^2}} \varphi + \frac{1}{p+1} \varphi^{p+1} \Big]$$

where p is any prime number, $\Box = -\partial_t^2 + \nabla^2$ is the D-dimensional d'Alembertian and metric with signature $(- + ... +)$ (Freund, Witten, Frampton, Okada, ...) .

- The equation of motion is

$$p^{-\frac{\Box}{2m_p^2}}\varphi = \varphi^p$$

It has trivial solutions $\varphi = 0$ and $\varphi = 1$. There are also inhomogeneous solutions resembling solitons. This equation separates in arguments and for any spatial direction x^i one has

$$\varphi(x^i) = p^{\frac{1}{2(p-1)}} \exp\Big(-\frac{p-1}{2m_p^2\, p\ln p}(x^i)^2\Big)$$

- Vladimirov investigation of nonlinear nonlocal equations of motions for *p*-adic scalar strings:
  1. investigation of solutions for open *p*-adic strings
  2. investigation of solutions for open-closed *p*-adic strings

$$\psi^{p^2} = e^{-\Box/4}\psi, \qquad \phi^p \psi^{p(p-1)/2} = e^{-\Box/2}\phi$$

where $\psi(t,x)$ and $\phi(t,x)$, $x = (x_1, x_2, \ldots, x_{d-1})$ are tachyonic fields for closed and open strings,respectively, and $\Box$ is d-dimensional d'Alembert operator.

Further developments of some Vladimirov investigations

- Developments of p-adic quantum mechanics
- Foundation of adelic quantum mechanics
- p-Adic and adelic gravity and cosmology, in particular in quantum cosmology
- Applications of Vladimirov p-adic pseudodifferential operator in wavelets (Kozyrev, Khrennikov,...) and protein dynamics (Avetisov, ...) .

- Adelic quantum mechanics can be viewed as a triple $(L_2(\mathbb{A}), W, U(t))$, where $L_2(\mathbb{A})$ is the Hilbert space on adelic space $\mathbb{A}$, W means Weyl quantization, and $U(t)$ is unitary evolution operator on $L_2(\mathbb{A})$.
- Adelic harmonic oscillator is related to the Riemann zeta function. Its simplest vacuum state is $\psi_0(x) = 2^{1/4} e^{-\pi x_\infty^2} \prod_p \Omega(|x_p|_p)$ and in momentum space $\tilde{\psi}_0(k) = 2^{1/4} e^{-\pi k_\infty^2} \prod_p \Omega(|k_p|_p)$. Using Mellin transform and Tate formula one obtains

$$\pi^{-\frac{s}{2}} \Gamma(\frac{s}{2}) \zeta(s) = \pi^{\frac{s-1}{2}} \Gamma(\frac{1-s}{2}) \zeta(1-s)$$

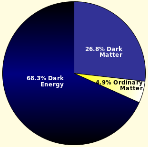
- Adelic quantum cosmology is an application of adelic quantum mechanics to minisuperspace cosmological models of the very early Universe.
- Adelic wave function for vacuum state of the de Sitter (and some other) model contains discreteness of space and time at the Planck length

$$\Psi(q) = \Psi_\infty(q) \prod_p \Omega(|q|_p) = \begin{cases} \Psi_\infty(q), & q \in \mathbb{Z} \\ 0, & q \in \mathbb{Q} \setminus \mathbb{Z}. \end{cases}$$

It means $q = n \cdot \ell_P$, where $\ell_P = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-33} cm$.

Possible *p*-adic origin of dark matter and dark energy

- If Einstein general theory of relativity is theory of gravity for the entire Universe then there is only about 5% of ordinary matter and 95% of matter in the Universe of unknown nature, i.e 95% of the Universe is dark.
- *Dark matter* (27%) should be responsible for rotation velocities of galaxies.
- *Dark energy* (68%) was introduced in 1998 as a possibility to explain Universe expansion acceleration.
- It is possible that there is some *p*-adic matter in the Universe, and that dark matter and dark energy have *p*-adic origin, i.e. origin in *p*-adic strings.



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