

COMPOSITION OPERATORS ON SOBOLEV SPACES IN A CARNOT GROUP

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Mainly we study mappings inducing composition operators on Sobolev spaces. In this talk we are going to present the basic notions regarding the problem under consideration. Moreover, we formulate our main result for isomorphic composition operators of Sobolev spaces on a Carnot Group. This talk is based on a joint work with Sergey Vodopyanov [2]. We develop and generalize ideas from the framework for $\mathbb{R}^n$, see [1].

A **Carnot group** $\mathbb{G}$ is a connected simply connected stratified nilpotent Lie group. This means that the Lie algebra $\mathfrak{g}$ of the group $\mathbb{G}$ admits a nilpotent stratification: $\mathfrak{g} = V_1 \oplus \dots \oplus V_m$, and $[V_1, V_j] = V_{j+1}$ for $j = 1, \dots, m-1$, whereas $[V_1, V_m] = \{0\}$. Let $X_1, \dots, X_n$ be vector fields constituting a basis of V_1 .

Sobolev space $L_p^1(D)$ consist of locally integrable functions $f: D \rightarrow \mathbb{R}$ with weak derivatives $X_i f \in L_p^1(D)$, $i = 1, \dots, n$. Let $\varphi: D \rightarrow D'$ is a measurable mapping and $L_q^1(D)$, $L_p^1(D')$ are Sobolev spaces on these domains. If a function $f \in L_p^1(D')$ is continuous then the composition $f \circ \varphi$ is well-defined almost everywhere on D . Assume that $f \circ \varphi \in L_q^1(D)$ and $\|f \circ \varphi\|_{L_q^1(D)} \leq K \|f\|_{L_p^1(D')}$ for all $f \in L_p^1(D') \cap C(D')$. Thus have just defined the **composition operator**:

$$L_p^1(D') \cap C(D') \ni f \mapsto \varphi^* f = f \circ \varphi \in L_q^1(D). \quad (1)$$

It is well known that operator (1) can be extended to the whole space $L_p^1(D')$ by the continuity.

Here we consider the case $p = q$ and the extension of φ^* is an isomorphism.

Theorem. *Let $p \geq 1$, $p \neq \nu$, and D, D' are domains on a Carnot group $\mathbb{G}$. Measurable mapping $\varphi: D \rightarrow D'$ induces an isomorphism of Sobolev spaces*

$$\varphi^*: L_p^1(D') \rightarrow L_p^1(D),$$

if and only if φ coincides almost everywhere with a quasi-isometric homeomorphism (w.r.t. Carnot Carathéodory distance) $\Phi: D \rightarrow \Phi(D)$ for which Sobolev spaces $L_p^1(\Phi(D))$ and $L_p^1(D')$ are equivalent.

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References

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2. S. K. Vodopyanov and N. A. Evseev. Isomorphisms of Sobolev spaces on Carnot groups and quasiisometric mappings. // Siberian Mathematical Journal, 2014, 5.