

GEODESICS AND TOPOLOGY OF HORIZONTAL-PATH SPACES IN CARNOT GROUPS

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On a sub-Riemannian manifold it is interesting to study the topology of the space of horizontal curves joining two points (the nonholonomic loop space); by applying Morse theory we can relate its topology with the structure of geodesics (critical points of the energy). Precisely we study the case where the points are 'infinitesimally close', in order to get the properties depending on the local structure of the distribution and avoiding properties due to the topology of the manifold.

This means that we focus on local models of sub-Riemannian manifolds, namely Carnot groups. We find the structure of the geodesics joining the origin with so called vertical points, where the most typical behaviour of nonholonomic constraints appear. Moreover, even though the space of horizontal paths is contractible, we measure its complexity by looking how the topology of sublevels of the Energy change, in the spirit of Morse theory.

References

1. *A. A. Agrachev, A. Gentile, A. Lerario*, Geodesics and admissible path spaces in Carnot groups, arXiv:1311.6727