

DIFFUSION BY OPTIMAL TRANSPORT IN THE HEISENBERG GROUP

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In this talk, we will consider the hypoelliptic diffusion, the “heat diffusion” of the subRiemannian Heisenberg group $\mathbb{H}$. We will show that in the Wasserstein space $\mathcal{P}_2(\mathbb{H})$, the space of probability measures with finite second moment, it is a curve driven by the gradient flow of the Boltzmann entropy, $\text{Ent}: \mathcal{P}_2 \rightarrow \mathbb{R} \cup \{\infty\}$. Conversely any gradient flow curve of Ent satisfies the hypoelliptic heat equation.

This illustrates and completes the theory of Ambrosio, Gigli and Savaré about the gradient flows of Ent on the Wasserstein spaces of some very general metric spaces.

References

1. *Luigi Ambrosio, Nicola Gigli, Giuseppe Savaré*. Calculus and heat flow in metric measure spaces and applications to spaces with Ricci bounds from below
Inventiones Mathematicae, to appear (2013).
2. *Nicolas Juillet*. Diffusion by optimal transport in Heisenberg groups.
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