Laplacian Flow of G_2-Structures on $S^3 \times \mathbb{R}^4$

Hazhgaly Kozhasov

Novosibirsk State University, Novosibirsk, Russia

hazhgaly@gmail.com

A 7-dimensional smooth manifold M admits a G_2-structure if there is a reduction of the structure group of its frame bundle from $GL(7,\mathbb{R})$ to the group G_2, viewed as a subgroup of $SO(7,\mathbb{R})$. On a manifold with G_2-structure there exists a "non-degenerate" 3-form φ, which determines a Riemannian metric g_φ in a non-linear fashion. Let (M,φ) be a manifold with G_2-structure. If φ is parallel with respect to Levi-Civita connection of the metric g_φ, $\nabla\varphi = 0$, then (M,φ) is called G_2-manifold. Such manifolds are always Ricci-flat and have holonomy contained in G_2. The condition $\nabla\varphi = 0$ is equivallent to φ to be closed, $d\varphi = 0$, and co-closed, $\delta\varphi = 0$, form. It is very interesting to understand how we can get a parallel φ on a certain manifold with G_2-structure via the evolution of some specific quantities. I will tell about the flow $\frac{\partial\varphi(t)}{\partial t} = \Delta\varphi$ on a $S^3 \times \mathbb{R}^4$, where $\varphi(t)$ is a continuous family of G_2-structures defined on this space and $\Delta = d\delta + \delta d$ is a Hodge-Laplacian operator.