

CONSTRUCTION OF CLASSICAL METRICS WITH SPECIAL HOLONOMIES VIA GEOMETRICAL FLOWS

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Many well-known metrics with curvature restrictions (such as special holonomy) have form $\bar{g} = dt^2 + g(t)$, where the metric $g(t)$ is usually a deformed metric on some well-studied space, for example homogenous space. The deformed metric $g(t)$ depends on the functions of variable t and the curvature restrictions which are the equations on the Riemannian or Ricci tensors instead of being partial differential equations become a ordinary differential equations. A class of interesting metrics appears when $\bar{g}$ is a cone metric $dt^2 + t^2 ds^2$. If one wants to get a flow that gives a constant curvature metric $\bar{g}$, he will be led to the flow

$$\frac{\partial}{\partial t} g = \sqrt{\text{Ric} - 4K}.$$

We call this flow the *Dirac flow*. Although the right-hand side of this flow is pseudodifferential operator of the first order the qualitative behavior of this flow is similar to the behavior of Ricci flow (at least for the simplest case of conformally round metric on S^3). This flow collapses the 3-dimensional sphere, such behavior at the origin $t = 0$ is so-called "nut"-type singularity. But some important metrics (e.g. Eguchi-Hanson metric) have the different type of singularity — the "bolt"-type — when only 1-dimensional circle in the Hopf bundle of S^3 is collapsed. To describe such metrics one is led to the flows with much more unpleasant right-hand side. For example, if the metric $g(t)$ satisfies the flow

$$\frac{\partial}{\partial t} g = \frac{1}{2} \sqrt{\det(\text{Ric})} \text{Ric}^{-1}.$$

then the metric $\bar{g}$ will be Eguchi-Hanson metric for appropriate initial data.